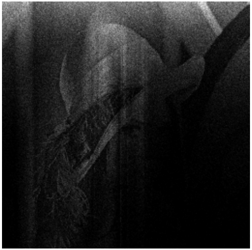

Multiview Attenuation Estimation and Correction.

Valentin Debarnot, Jonas Kahn and Pierre Weiss

27/06/2017

ITAV - IMT

Motivating example - Lidar (1D)

The lidar equation

$$u_1(x) = \beta(x) \exp\left(-\int_0^x \alpha(t)\, dt\right)$$

- α is the **attenuation** (absorption or extinction coefficient).
- β is the **density** (backscatter coefficient).

The lidar inverse problem

Retrieve α and β from u_1.

Motivating example - Lidar (1D)

Standard resolution

- Assuming that $\alpha = \beta^\gamma$, a popular method is Klett's estimator.

James D Klett.
Stable analytical inversion solution for processing lidar returns.
Applied Optics, 20(2):211–220, 1981.

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Problem

Both approaches assume direct or indirect knowledge of density β.

Motivating example - Lidar (1D)

A tempting idea: use two opposite lidars

Gerard J Kunz.
Bipath method as a way to measure the spatial backscatter and extinction coefficients with lidar.
Applied optics, 26(5):794–795, 1987.

Juan Cuesta and Pierre H Flamant.
Lidar beams in opposite directions for quality assessment of Cloud-Aerosol Lidar with Orthogonal Polarization spaceborne measurements.
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Analytical inversion

$$u_1(x) = \beta(x) \exp\left(-\int_0^x \alpha(t)\, dt\right) \text{ and } u_2(x) = \beta(x) \exp\left(-\int_x^1 \alpha(t)\, dt\right)$$

Setting

$$v(x) = \log\left(\frac{u_2(x)}{u_1(x)}\right),$$

we get:

$$\alpha(x) = \frac{1}{2}\frac{\partial}{\partial_x} v(x) \quad \text{and} \quad \beta(x) = \frac{u_1(x)}{\exp\left(-\int_0^x \alpha(t)\, dt\right)}.$$

Motivating example - Lidar (1D)

Problems

- Density should satisfy $\beta(x) > 0$ for all x.
- Inversion is **extremely** unstable to noise.
- Existing solvers are very basic filtering approaches.

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- Inversion is **extremely** unstable to noise.
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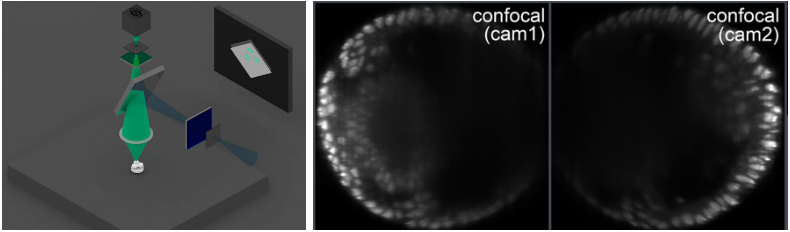
Objectives of this work

- Extend the principle to a larger setting.
- Provide a good statistical estimator.

Extension to fluorescence microscopy - Confocal



A CONFOCAL MICROSCOPE.

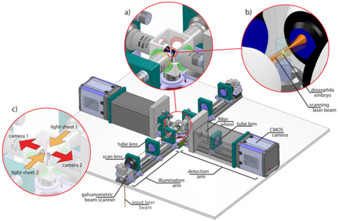
A similar problem

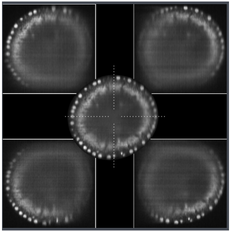
By using 4-Pi confocal microscopes, we get:

$$u_1 = \beta \exp\left(-A_1 \alpha\right) \text{ and } u_2 = \beta \exp\left(-A_2 \alpha\right)$$

where A_1 and A_2 are integral operators (e.g. primitive operators).

Extension to fluorescence microscopy - SPIM





A MULTI-VIEW SPIM MICROSCOPE.

A similar problem (again)

In general, we get a sequence of signals:

$$u_i = \beta \exp(-A_i \alpha), 1 \leq i \leq m$$

The problem is to recover α and β from the sequence (u_i).

A statistical estimator and its computation

Discretized model

We assume that

$$u_i = \mathcal{P}\left(\beta \exp(-A_i \alpha)\right),$$

where

- $\mathcal{P}(z)$ denotes a Poisson distributed random variable of parameter z.
- $A_i \in \mathbb{R}^{n\times n}$ is a discretized integral operator.

Maximum A Posteriori (MAP) estimator

$$\max_{\alpha\in\mathbb{R}^n, \beta\in\mathbb{R}^n} \mathbb{P}\left(\alpha, \beta | (u_i)\right)$$

Assuming independence of α and β:

$$\min_{\alpha\in\mathbb{R}^n, \beta\in\mathbb{R}^n} F(\alpha, \beta)$$

$$F = \left\langle \sum_{j=1}^m [\exp(-(A_j\alpha))\beta + u_j((A_j\alpha) - \log(\beta))], \mathbb{1} \right\rangle + R_\alpha(\alpha) + R_\beta(\beta)$$

A statistical estimator and its computation

Proposition

Function $(\alpha, \beta) \in \mathbb{R}^n \times \mathbb{R}^n \mapsto F(\alpha, \beta)$ is nonconvex on $\mathbb{R}^n \times \mathbb{R}^n$.

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Proposition - Convexifying trick

Set $R_\beta(\beta) = \mathbb{1}_{\mathbb{R}^n_+}(\beta)$, then the optimal β satisfies

$$\beta = \beta(\alpha) = \frac{\sum_{i=1}^m u_i}{\sum_{i=1}^m \exp\left(-A_i \alpha\right)}$$

If R_α is convex, then $G(\alpha) = F(\alpha, \beta(\alpha))$ is a **convex** function.

A statistical estimator and its computation

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Recovering the attenuation boils down to a **convex program**:

$$\hat{\alpha} = \operatorname*{argmin}_{\alpha \in \mathbb{R}^n} \sum_{i=1}^n \sum_{j=1}^m u_j[i] \left[(A_j \alpha)[i] + \log \left(\sum_{j=1}^m \exp(-(A_j \alpha)[i]) \right) \right] + R_\alpha(\alpha).$$

Estimating the attenuation

A numerical challenge...

$$\hat{\alpha} = \operatorname*{argmin}_{\alpha \in \mathbb{R}^n} \sum_{i=1}^{n} \sum_{j=1}^{m} u_j[i] \left[(A_j \alpha)[i] + \log \left(\sum_{j=1}^{m} \exp(-(A_j \alpha)[i]) \right) \right] + R_\alpha(\alpha),$$

with $R_\alpha(\alpha) = \|\nabla \alpha\|_1$ the total variation.

A natural solver: the ADMM

Pierre-Louis Lions and Bertrand Mercier.
Splitting algorithms for the sum of two nonlinear operators.
SIAM Journal on Numerical Analysis, 16(6):964–979, 1979.

Michel Fortin and Roland Glowinski.
Augmented Lagrangian methods: applications to the numerical solution of boundary-value problems, volume 15.
Elsevier, 2000.

Estimating the attenuation

Numerical challenges

- R_α non-differentiable.
- With $A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 0 & \dots & 0 \\ & & & \ddots & & \\ \dots & \dots & \dots & & \dots & 0 \\ 1 & 1 & 1 & 1 & \dots & 1 \end{pmatrix}$, $\|A_1\|_{2\to 2} \geq n$.
- Proximal operator of logsumexp (in dimension m):

$$\operatorname{prox}_{\gamma \operatorname{logsumexp}}(z) = \underset{x \in \mathbb{R}^n}{\operatorname{argmin}} \, \gamma \sum_{j=1}^{2} u_j[i] \log \left(\sum_{j=1}^{m} \exp(-(x_j)[i]) \right) + \frac{1}{2} \|x - z\|_2^2.$$

- Linear systems are solved with preconditioned conjugate gradient.

Estimating the attenuation

Numerical challenges

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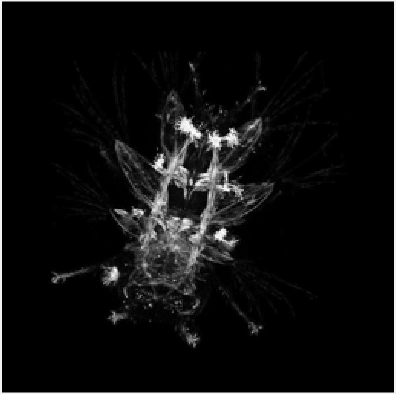
- Linear systems are solved with preconditioned conjugate gradient.

Limit

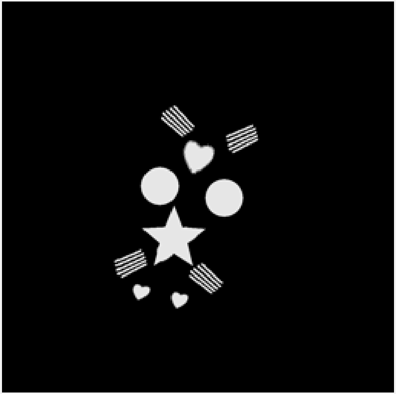
Proximal operator of logsumexp only done in dimension 2.

Estimating the attenuation

Data to recover



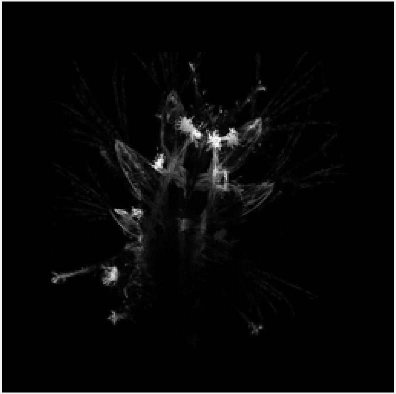
Density β



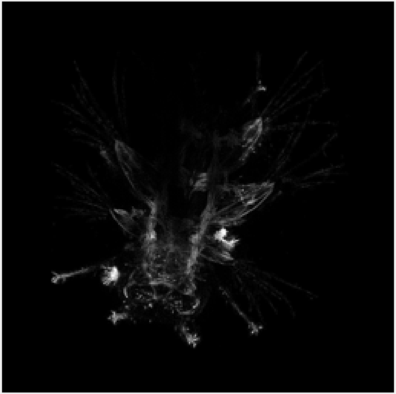
Attenuation α

Estimating the attenuation

Observations



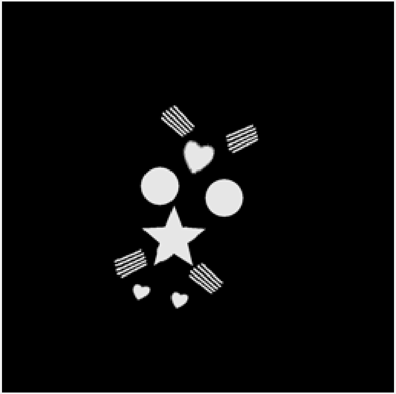
Observation 1



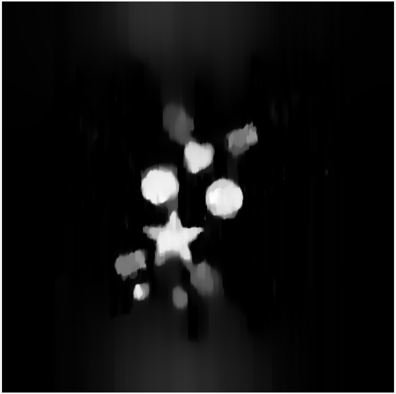
Observation 2

Estimating the attenuation

Data to recover



True attenuation



Estimated attenuation

Estimating the density

A noisy density

With $R_\beta(\beta) = \mathbb{1}_{\mathbb{R}^n_+}(\beta)$:

$$\beta = \frac{\sum_{i=1}^m u_i}{\sum_{i=1}^m \exp\left(-A_i \alpha\right)}$$

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With $R_\beta(\beta) = \mathbb{1}_{\mathbb{R}_+^n}(\beta)$:

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MAP estimator

With a general $R_\beta(\beta)$, the MAP estimator of β solves:

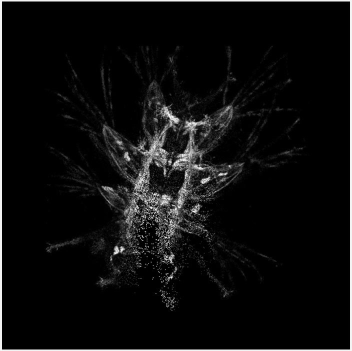
$$\hat{\beta} = \underset{\beta \in \mathbb{R}_+^n}{\operatorname{argmin}} \left\langle \sum_{j=1}^m \beta \exp\left(-A_j \hat{\alpha}\right) - u_j \log\left(\beta\right), \mathbb{1} \right\rangle + R_\beta(\beta),$$

A natural solver

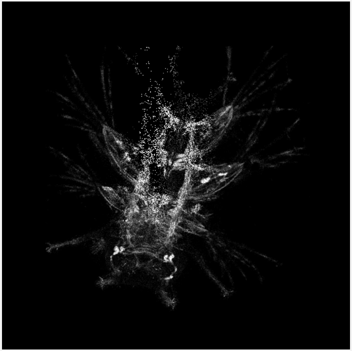
ADMM algorithm with R_β the total variation.

Correcting the attenuation

After direct inversion



Direct inversion 1



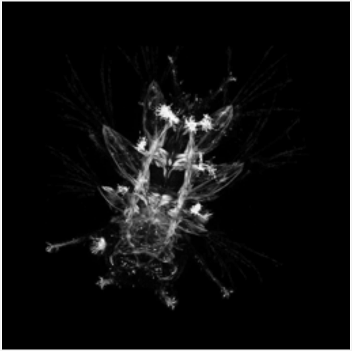
Direct inversion 2

Direct inversion

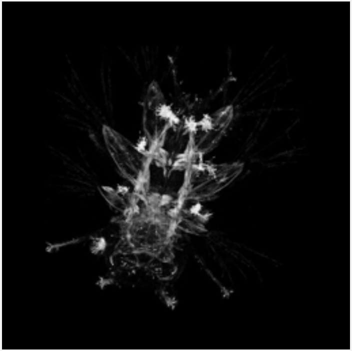
$$\tilde{\beta}_i = \frac{u_i}{\exp\left(-A_i \alpha\right)}, \ i \in \{1, 2\}.$$

Correcting the attenuation

Multi-view denoising result



True density



Recovered density

Estimator – Multiple-heteroskedastic denoising

$$\min_{\beta \in \mathbb{R}^n} \sum_{i=1}^{n} \sum_{j=1}^{m} [\exp(-(A_j \alpha)[i]) \beta[i] - u_j[i] \log(\beta[i])] + R_\beta(\beta).$$

A last numerical experiment

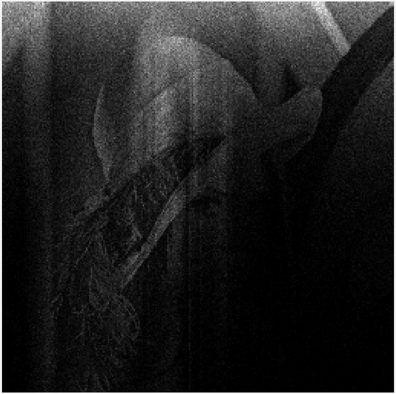
Data to recover

Density β

Attenuation α

A last numerical experiment

Observations

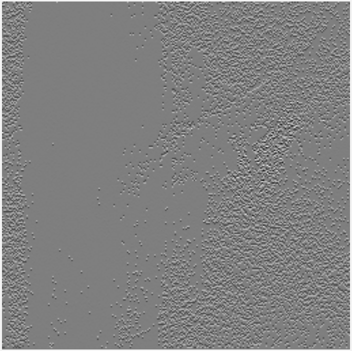


Observation 1

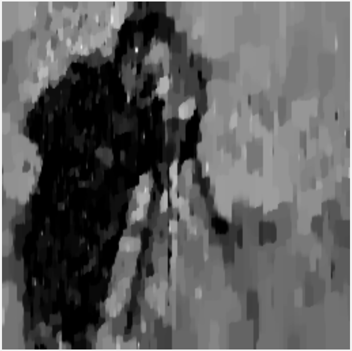
Observation 2

A last numerical experiment

Data to recover



Direct estimate



Regularized estimate

Direct estimate

$$\alpha = \frac{1}{2} \frac{\partial}{\partial_x} \left[\log \left(\frac{u_2}{u_1} \right) \right].$$

A last numerical experiment

Data to recover

MLE estimate

Regularized estimate

Direct inversion

$$\tilde{\beta} = \frac{\sum_{i=1}^{m} u_i}{\sum_{i=1}^{m} \exp\left(-A_i \alpha\right)}.$$

Conclusion

Positive aspects

- Attenuation may provide useful information.
- Designed efficient and robust numerical estimators.
- Seems realistic enough to treat some real data.

Perspectives

- Ongoing work on lidar data.
- Taking into account diffraction and scattering.

More details

Valentin Debarnot, Jonas Kahn, and Pierre Weiss.
Multiview Attenuation Estimation and Computation.
arXiv preprint arXiv:1701.02615, 2017.