Numerical optimization and its applications

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Nonsmooth, nonconvex optimization with applications to polynomial and eigenvalue optimization

Michael L. Overton, Courant Institute of Mathematical Sciences, NYU

In many applications one wishes to minimize an objective function that is not convex and is not differentiable at its minimizers. We start by discussing two algorithms for minimization of nonsmooth, nonconvex functions. Gradient Sampling is an easily explained method that, although computationally intensive, has a nice convergence theory that we present at a high level. The method is robust and the convergence theory has been extended to constrained problems. The Broyden-Fletcher-Goldbarb-Shanno (BFGS) quasi-Newton method was developed in 1970 for smooth problems, but it is remarkably effective for nonsmooth problems too. Although our theoretical results for BFGS in the nonsmooth case are quite limited, we have made some remarkable empirical observations and have had broad success in applications. We present some of these in detail. Limited Memory BFGS is a popular extension for large problems, and it is also applicable to the nonsmooth case, although our experience with it is more mixed.

We then turn to some specific applications. Suppose that the coefficients of a monic polynomial or entries of a square matrix depend affinely on parameters, and consider the problem of minimizing the root radius (maximum of the moduli of the roots) or root abscissa (maximum of their real parts) in the polynomial case and the spectral radius or spectral abscissa in the matrix case. These functions are not convex and they are typically not locally Lipschitz near minimizers. We first address polynomials, for which some remarkable analytical results are available in one special case, and then consider the more general case of matrices, focusing on the static output feedback problem arising in control of linear dynamical systems. We also briefly discuss some spectral radius optimization problems arising in the analysis of the transient behavior of a Markov chain and the design of smooth surfaces using subdivision algorithms.

Bundle methods for nonsmooth optimization

Dominikus Noll, Institut de Mathématiques de Toulouse

Nonsmooth optimization problems arise in a variety of applications. This includes Lagrangian relaxation for difficult mixed integer programming problems, large-scale stochastic programming, bi-level programming, but also mechanical contact problems and various applications in control.

Bundle algorithms have always been a prominent class of methods to address nonsmooth optimization problems. In this lecture we show how the bundle approach can be successfully extended to nonconvex problems. We discuss convergence theory, but also practical and implementational issues like stopping, solving the tangent program, computing subgradients, dealing with inexact subgradients and function values, and much else.

Numerical methods for complementarity problems and applications

Mounir Haddou, INSA de Rennes

In this part of the course we will review some results on linear complementarity problems (LCP). We will see in particular the different classes of methods such as: interior point methods, semismooth methods, projective methods and regularization methods. We will also focus on nonlinear complementarity problems (NCP), AVP problems (a more general class than LCP) and finally on optimization problems with equilibrium constraints.

Much of the course will be devoted to the implementation of several applications, we will also see some simple heuristics to solve some discrete optimization problems (optimal partition problems and minimization cardinality)