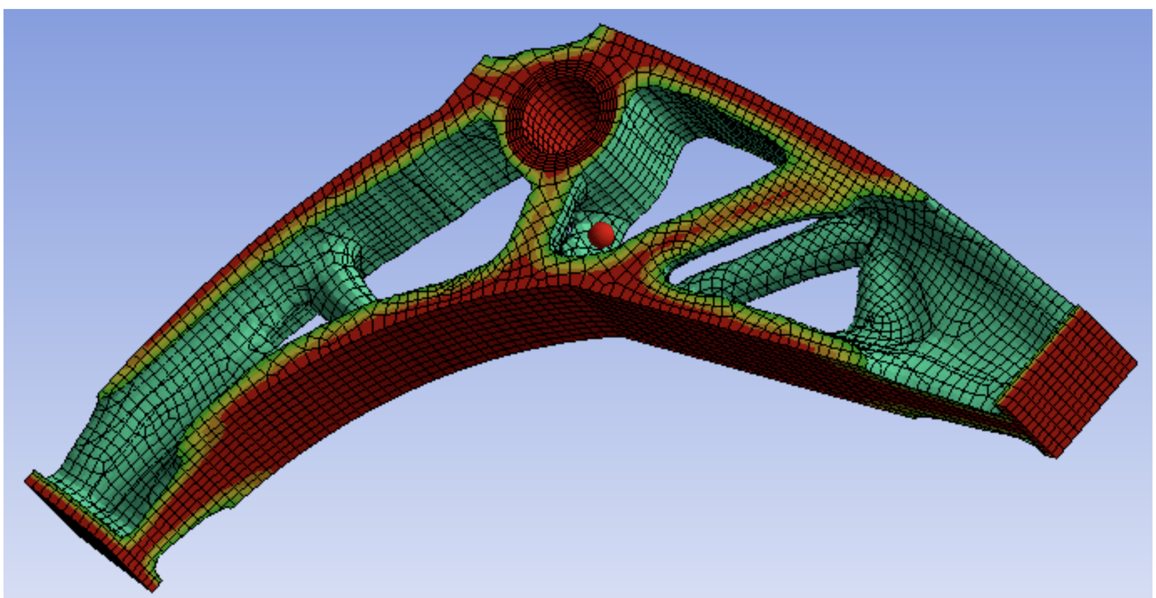
Journées Annuelles du GdR MOA

(Mathématiques de l'Optimisation et Applications)

Pau, 17–19 octobre 2018



Programme des journées

Mercredi 17 octobre 2018

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13h30-14h00 : Accueil

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14h00-15h00 : Conférence plénière de **Radu Ioan Boţ**

Proximal algorithms for nonconvex and nonsmooth optimization problems

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15h00-15h30 : Pause café et discussions

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15h30-16h00 : Exposé de **Vassilis Apidopoulos**

Descente de gradient inertielle sous conditions géometriques

16h00-16h30 : Exposé de **Nang Thieu Nguyen**

A convergence result for a vibro-impact problems with a nonconvex moving set of constraints

16h30-17h00 : Exposé de **Walter Cedric Simo Tao Lee**

Principle de Morozov via la Dualité de Lagrange

17h00-17h30 : Exposé de **Michel Thera**

Some new developments on the Campanato nearness condition

Jeudi 18 octobre 2018

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08h30-09h00 : Exposé de **Ngoc Nguyen Tran**

Local analysis of a regularized primal-dual algorithm for nonlinear programming without constraint qualification

09h00-09h30 : Exposé de **Florent Nacry**

Processus de Moreau à variation tronquée

09h30-10h00 : Exposé de **Beniamin Bogosel**

Optimisation paramétrique de formes en utilisant la fonction support

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10h00-10h30 : Pause café et discussions

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10h30-11h30 : Conférence plénière de **Aude Rondepierre**

Global probability of collision for space encounters : problem modeling via occupation measures

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12h00-14h00 : Repas à "La Vague"

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14h00-15h00 : Conférence plénière de **Aris Daniilidis**

On functions that saturate the Clarke subdifferential

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15h00-15h30 : Pause café et discussions

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15h30-16h00 : Exposé de **Vincenzo Basco**

Necessary conditions for infinite horizon optimal control problems under state constraints and Hamilton-Jacobi-Bellman equations

16h00-16h30 : Exposé de **Fatima Zahra Tani**

Contrôle périodique : généralités et applications

16h30-17h00 : Exposé de **Jean-Baptiste Hiriart-Urruty**

Mathématiciens élus politiques : quelques exemples

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20h30 : Diner au restaurant "La Belle Époque"

Vendredi 19 octobre 2018

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09h00-09h30 : Exposé de **David Gaudrie**

Achievable Goals in Bayesian Multi-Objective Optimization

09h30-10h00 : Exposé de **Sebastián Tapia**

Selfcontracted curves, applications and extensions

10h00-10h30 : Exposé de **Michel De Lara**

Dual Problems For Exact Sparse Optimization

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10h30-11h00 : Pause café et discussions

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11h00-12h00 : Conférence plénière de **Alexandra Schwartz**

A complementarity-based approach to cardinality-constrained optimization

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12h00-12h15 : Clôture

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12h30-14h00 : Repas à “La Vague”

Résumés des exposés

Descente de gradient inertielle sous conditions géometriques

Vassilis Apidopoulos
IMB, Université de Bordeaux, France

Jean-François Aujol
IMB, Université de Bordeaux, France

Charles Dossal
IMT, Université de Toulouse 3 – Paul Sabatier, France

Aude Rondepierre
IMT, Université de Toulouse 3 – Paul Sabatier, France

Mots-clefs : Optimisation convexe et lisse, algorithme de descente de gradient inertielle, conditions de Łojasiewitcz, forte convexité, comportement asymptotique

Dans cet exposé on présentera une version inertielle de l'algorithme de descente de gradient "à la Nesterov". En particulier on recherche les différentes propriétés de convergence pour cet algorithme, en fonction de la géometrie de la fonction minimisante et du paramètre d'inertie. Cette étude est en parallèle avec l'étude en version continue, faite dans [3]. On compare également les résultats avec d'autres versions inertielles de l'algorithme de descente de gradient.

Références

[1] V. Apidopoulos, J-F Aujol and C. Dossal, *Convergence rate of inertial Forward-Backward algorithm beyond Nesterov's rule*, HAL preprint, hal-01551873, 2017.

[2] H. Attouch and A. Cabot, *Convergence rates of inertial forward-backward algorithms*, SIAM Journal on Optimization, 2018.

[3] J-F. Aujol, C. Dossal and A. Rondepierre, *Optimal convergence rates for Nesterov acceleration*, arXiv preprint arXiv :1805.05719, 2018.

Necessary conditions for infinite horizon optimal control problems under state constraints and Hamilton-Jacobi-Bellman equations

Vincenzo Basco
IMJ-PRG, Sorbonne Université, France

Mots-clefs : Optimal control ; Infinite horizon ; State constraints ; Hamilton-Jacobi-Bellman equations.

In this talk I will discuss sufficient conditions for Lipschitz regularity of the value function for an infinite horizon optimal control problem subject to state constraints. I focus on problems with cost functional admitting a discount rate factor and allow time dependent dynamics and Lagrangian. Furthermore, state constraints may be unbounded and may have a nonsmooth boundary. Lipschitz regularity is recovered as a consequence of estimates on the distance of a given trajectory of control system from the set of all its viable (feasible) trajectories, provided the discount rate is sufficiently large (cfr. [2]). I will talk about first order necessary optimality conditions : a constrained maximum principle and sensitivity relations involving generalized gradients of the value function (cfr. [1]). Finally, I will address nonautonomous Hamilton-Jacobi-Bellman equations, with time-measurable data, and their weak solutions : an existence and uniqueness result for solutions to the H-J-B equation associated with an infinite horizon control problem is discussed (cfr. [3]).

Références

[1] V. Basco, P. Cannarsa and H. Frankowska, *Necessary conditions for infinite horizon optimal control problems with state constraints*, Mathematical Control & Related Fields, 8(3&4) :535-555, 2018.

[2] V. Basco and H. Frankowska, *Lipschitz continuity of the value function for the infinite horizon optimal control problem under state constraints*, Submitted for publication.

[3] V. Basco and H. Frankowska, *Hamilton-Jacobi-Bellman Equations with Time-Measurable Data and Infinite Horizon*, Submitted for publication.

Optimisation paramétrique de formes en utilisant la fonction support

Pedro R.S. Antunes
GMP, Université de Lisbon, Portugal

Beniamin Bogosel
CMAP, École Polytechnique, France

Mots-clefs : optimisation de formes, fonction support, contraintes non-locales

L'étude numérique des problèmes d'optimisation de forme sous contraintes de convexité, diamètre ou largeur constante est difficile. La source des difficultés est la non-localité de ces contraintes : la classe des perturbations admissibles est restreinte aux endroits ou la contrainte est saturée. Dans cet exposé on présente une méthode, basée sur la fonction support associée à un ensemble convexe, qui permet de contourner ces difficultés. Les contraintes sont transformées dans des inégalités algébriques sur les coefficients d'une décomposition spectrale de la fonction support. Ceci permet l'utilisation des codes standard en optimisation pour approcher les solutions d'une famille assez large de problèmes d'optimisation de formes.

On considère l'optimisation des fonctionnelles variées qui dépendent du volume, du périmètre et des valeurs propres de l'opérateur Dirichlet-Laplace sous différentes contraintes évoquées plus haut. En particulier, on confirme numériquement la conjecture de Meissner concernant les corps de largeur constante de volume minimal en dimension trois, par résolution directe d'un problème d'optimisation.

Références

[1] P.R.S Antunes and B. Bogosel, *Parametric shape optimization using the support function*, 2018, preprint arXiv :1809.00254

Proximal algorithms for nonconvex and nonsmooth optimization problems

Radu Ioan Boţ
University of Vienna, Austria

In the first part we address the minimization of the sum of a proper and lower semicontinuous function with a possibly nonconvex smooth function by means of a proximal-gradient algorithm with inertial and memory effects. We prove that the sequence of iterates converges to a critical point of the objective, provided that a regularization of the latter function is a so-called KL function ; in other words, it satisfies the Kurdyka-Łojasiewicz inequality. To the class of KL functions belong semialgebraic, real subanalytic, uniformly convex and convex functions satisfying a growth condition.
In the second part we propose a proximal algorithm for the minimization of objective functions consisting of three summands : the composition of a non- smooth function with a linear operator, another nonsmooth function, each of the nonsmooth summands depending on an independent block variable, and a smooth function which couples the two block variables. The algorithm is a full splitting method, which means that the nonsmooth functions are processed via their proximal operators, the smooth function via gradient steps, and the linear operator via matrix times vector multiplication. We provide sufficient conditions for the boundedness of the generated sequence and prove that any cluster point of the latter is a KKT point of the minimization problem. In the setting of the Kurdyka-Łojasiewicz property we show global convergence, and derive convergence rates for the iterates in terms of the Łojasiewicz exponent.

Dual Problems For Exact Sparse Optimization

Aris Daniilidis
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Gonzalo Flores
University of Chile, Chile

Mots-clefs : Lipschitz function, maximal Clarke subdifferential, lineability, spaceability

We prove that the set of Lipschitz functions with maximal Clarke subdifferential at every point contains a linear subspace of uncountable dimension. Our approach is constructive and in contrast to a previous result of similar flavour, by J. Borwein and X. Wang, it does not relate to the Baire category theorem. In particular we establish lineability (and spaceability for the Lipschitz norm) of the above set inside the set of all Lipschitz continuous functions.

Références

[1] A. Daniilidis and G. Flores, *Linear structure of functions with maximal Clarke subdifferential*, preprint, 2018.

Dual Problems For Exact Sparse Optimization

Jean-Philippe Chancelier
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Michel De Lara
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Mots-clefs : sparse optimization, l_0 “norm”, Fenchel-Moreau conjugacy

Exact sparse optimization problems (also known as sparsity-constrained problems) can be formulated as the minimization of a criterion under a constraint that the l_0 “norm” be less than a given integer, that measures the sparsity of the solution. Since the l_0 “norm” is not convex, such problems do not generally display convexity properties, even if the criterion to minimize is convex.

One route to attack such problems is to replace the sparsity constraint by a convex penalizing term, that will induce sparsity [2, 1]. Thus doing, we lose the original exact sparse optimization problems, but we gain convexity (benefiting especially of duality tools with the Fenchel conjugacy).

We propose another route, where we lose convexity but where we gain at keeping the original exact sparse optimization formulation. For this purpose, we introduce an adapted conjugacy, induced by a novel coupling [4, 3], the Fenchel coupling after primal normalization. This coupling has the property of being constant along primal rays, like the l_0 “norm”. Thus equipped, we present a way to build a dual problem, that is a lower bound of the original exact sparse optimization problem. We illustrate our result on the classical least squares regression sparse optimization problem.

Références

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[3] J. E. Martínez-Legaz, *Generalized convex duality and its economic applications*, In S. S. Hadjisavvas N., Komlósi S., editor, Handbook of Generalized Convexity and Generalized Monotonicity. Nonconvex Optimization and Its Applications, volume 76, pages 237–292. Springer Verlag, 2005.

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Achievable Goals in Bayesian Multi-Objective Optimization

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Rodolphe le Riche
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Victor Picheny
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Mots-clefs : Bayesian Optimization, Computer Experiments, Multi-Objective Optimization

We consider multi-objective optimization problems, $\min_{\mathbf{x}\in\mathbb{R}^d}(f_1(\mathbf{x}),\ldots,f_m(\mathbf{x}))$, where the functions are expensive to evaluate. In such a context, Bayesian methods relying on Gaussian Processes (GP) [1], adapted to multi-objective problems [2] have allowed to approximate Pareto fronts in a limited number of iterations.

In the current work, we assume that the Pareto front center has already been attained (typically with the approach described in [3]) and that a computational budget remains. The goal is to uncover of a broader central part of the Pareto front : the intersection of it with some region to target, $\mathcal{I}_{\mathbf{R}}$ (see Fig. 1). $\mathcal{I}_{\mathbf{R}}$ has however to be defined carefully : choosing it too wide, i.e. too ambitious with regard to the remaining budget, will lead to a non converged approximation front. Conversely, a suboptimal diversity of Pareto optimal solutions will be obtained if choosing a too narrow area.

The GPs allow to forecast the future behavior of the algorithm : they are used in lieu of the true functions to anticipate which inputs/outputs will be obtained when targeting growing parts of the Pareto front. Virtual final Pareto fronts corresponding to a possible version of the approximation front at the depletion of the budget are produced for each $\mathcal{I}_{\mathbf{R}}$. A measure of uncertainty is defined and applied to all of them to determine the optimal improvement region $\mathcal{I}_{\mathbf{R}^*}$, balancing the size of the approximation front and the convergence to the Pareto front.

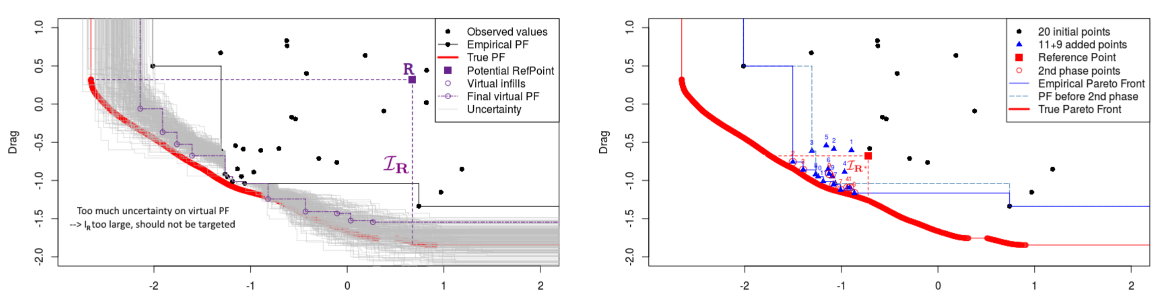

Figure 1 – Left : virtual Pareto front (dashed, purple) obtained if targeting $\mathcal{I}_{\mathbf{R}}$ for the 9 remaining iterations. The uncertainty (grey) at the end of the optimization is forecasted to be too large because $\mathcal{I}_{\mathbf{R}}$ is too wide. The optimal improvement region $\mathcal{I}_{\mathbf{R}^*}$ is shown on the right. Good convergence is obtained in this region.

Références

[1] D.R. Jones, M. Schonlau and W.J. Welch, *Efficient Global Optimization of expensive black-box functions*, Journal of Global Optimization, 13(4), 455-492.

[2] M. Emmerich, K.C. Giannakoglou and B. Naujoks, *Single-and multiobjective evolutionary optimization assisted by Gaussian random field metamodels*, Springer Series in Operations Research and Financial Engineering. IEEE Transactions on Evolutionary Computation, 10(4), 421-439.

[3] D. Gaudrie, R. Le Riche, V. Picheny, B. Enaux and V. Herbert, *Budgeted Multi-Objective Optimization with a Focus on the Central Part of the Pareto Front - Extended Version*, `arXiv` pre-print 1809.10482v1.

Mathématiciens élus politiques : quelques exemples

Jean-Baptiste Hiriart-Urruty
IMT, Université de Toulouse 3 – Paul Sabatier, France

"Les mathématiciens ne s'intéressent qu'à ce qu'ils font", "les "matheux" ne savent rien faire et ne font rien d'autre que des maths", "ils sont "bunkerisés" dans leur tour d'ivoire les profs de maths"..., autant d'appréciations acides que l'on entend parfois à propos des mathématiciens... Et pourtant, comme d'autres citoyens, ils participent à la vie publique et même politique, parfois à un haut niveau, comme cet article essaie de le montrer. Je ne parle pas de présidents d'établissements universitaires ou d'écoles d'ingénieurs, de conseillers ministériels éphémères, ou de conseillers municipaux, voire de maires de petites et moyennes villes... , où les collègues sont bien présents, mais de postes de plus haut niveau politique comme **élus** : maires de villes importantes, conseillers départementaux [1], députés, sénateurs, ou nommés comme secrétaires d'état ou ministres. J'ai recensé ici quelques cas sous forme de "vignettes", comme dans notre précédente publication sur les mathématiciens dans le patrimoine régional du "Grand Sud-Ouest" (voir [5]), et si votre réaction d'auditeur est : *"Oui mais il a oublié tel ou tel cas"*, ceci est une invite à compléter la liste par votre contribution.
Nous connaissons tous des engagements publics de mathématiciens, avec des prises de position proéminentes (par exemple Henri Cartan [2], Laurent Schwartz, Alexandre Grothendieck, ...) ; certains ont même payé de leur vie cet engagement (Maurice Audin (1932-1957), Mehdi Ben Barka (1920-1965), Ibni Oumar Saleh (1949-2008)).
Notre présentation ici est faite dans un cadre bien délimité que nous explicitons par les deux points ci-dessous :

- Une "fenêtre temporelle" qui se restreint au 20ème siècle et au début du 21ème [pour l'essentiel des activités scientifiques et politiques des mathématiciens considérés] ;
- Une définition de "mathématicien élu politique" satisfaisant les critères suivants : études mathématiques avancées (jusqu'au Master, Doctorat ou Agrégation) ; fonctions exercées de professeur de mathématiques ou d'enseignant-chercheur ; élu politique (conseiller départemental, député, maire de grande ville, président de région, ministre).

Le spectre des cas envisageables est très étendu. Habituellement, c'est un mathématicien qui, après avoir exercé un certain temps (comme professeur, enseignant-chercheur), embrasse une carrière politique, et ne revient plus aux mathématiques. Ce n'est pas toujours le cas, il y a eu des hommes politiques, maires de grandes villes par exemple, qui à un moment donné ont abandonné tout mandat d'élu et sont revenus simples professeurs.
Notre palette embrasse des situations très variées. Nous commençons par la France et sa partie "Grand Sud-Ouest", parcourons quelques pays d'Europe (Italie, Pologne, Allemagne, Espagne, Portugal), puis les Antilles, l'Afrique et le Moyen-Orient, l'Amérique latine... et terminons "en roue libre" avec quelques noms d'hommes politiques qui, s'ils ne peuvent être qualifiés de mathématiciens (au sens indiqué plus haut), ont eu une formation initiale en mathématiques, au moins jusqu'au niveau Licence compris.
Le texte complet est à paraître sur le site web **Culture*Math***.

1. Jusqu'en 2013, une fonction élective équivalente était celle de conseiller général.
2. Européen convaincu, H. Cartan (1904-2008), présida de 1974 à 1985 le Mouvement Fédéraliste Européen. En 1984, il fut, en France, à la tête d'une liste de candidats au parlement Européen, liste intitulée "Pour les Etats-Unis d'Europe". Sans vrais soutiens politiques ni financiers, cette liste ne recueillit que $0,4\%$ des suffrages exprimés.

Références

[1] A-L. Anizan, *Paul Painlevé et la guerre (1908-1933). Science poliade et défense nationale*, Des mathématiciens et des guerres ; Histoires de confrontations (19e-20e siècles), CNRS Editions (2013), 37-52.

[2] D. Bliss, *A mathematician runs for political office*, Notices of the American Mathematical Society, page 207 (February 2009).

[3] A. Durand, *Mathematicians and politics : new research scenarios*, Lettera Matematica, Vol. 4 (2017), 161-165.

[4] J-B. Hiriart-Urruty and H. Caussinus, *Sarrus, Borel, Deltheil - Le Rouergue et ses mathématiciens*, Gazette des Mathématiciens (de la SMF), n°104, 88-97 (2005).

[5] J-B. Hiriart-Urruty, *Les mathématiciens dans le patrimoine régional du "Grand Sud-Ouest"*, Site CultureMath (2018) : https ://culturemath.ens/fr.

[6] *Jornada matemática*, livre édité à l'occasion d'un congrès pour marquer l'année mondiale des mathématiques en 2000.

Processus de Moreau à variation tronquée

Florent Nacry
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Université de Montpellier, Institut Montpelliérain Alexander Grothendieck

Mots-clefs : Analyse variationnelle, processus de rafle de Moreau, cône normal, application absolument continue, distances de Hausdorff-Pompeiu tronquées.

Etant donnés un réel $T > 0$, $C : [0,T] \rightrightarrows \mathcal{H}$ une multi-application à valeurs convexes fermées non vides et $u_0 \in C(0)$, on s'intéresse aux applications $u : [0,T] \to \mathcal{H}$ absolument continues sur $[0,T]$ vérifiant

$$(SP)\begin{cases} -\dot{u}(t) \in N(C(t); u(t)) & \lambda\text{-p.p. } t \in [0,T], \\ u(t) \in C(t) \quad \text{pour tout } t \in [0,T], \\ u(T_0) = u_0, \end{cases}$$

où $N(\cdot;\cdot)$ désigne le *cône normal au sens de l'analyse convexe*. Cette inclusion différentielle introduite par J.J. Moreau en 1971 ([4]) est notamment connue pour sa grande variété d'applications (voir, par exemple, [1] et les références à l'intérieur). Ceci a naturellement conduit au développement de nombreuses variantes de (SP) : non-convexe, variation bornée, second ordre, stochastique, banachique, perturbé, avec dépendance de l'état, avec contrôle...
Le point commun des divers contextes mentionnés ci-dessus réside dans l'hypothèse d'un contrôle sur la variation (i.e., le mouvement) de l'ensemble mobile $C(\cdot)$ prenant la forme générique

$$\operatorname{haus}(C(s), C(t)) \le \mu(]s,t]) \quad \text{pour tout } s,t \in [0,T], \tag{1}$$

pour une certaine mesure positive μ sur $[0,T]$. Cette hypothèse (avec $\mu = \lambda$) garantit entres autres l'existence de solutions de (SP). Malheureusement, il s'avère ([8]) que de nombreux ensembles non bornés ne vérifient pas l'inégalité ci-dessus. Ceci amène à substituer dans (1) la distance de Hausdorff-Pompeiu $\operatorname{haus}(\cdot,\cdot)$ par des versions ρ-tronquées (où $\rho > 0$) $\operatorname{haus}_\rho(\cdot,\cdot)$ et $\widehat{\operatorname{haus}}_\rho(\cdot,\cdot)$ définies à travers les excès

$$\operatorname{exc}_\rho(S,S') := \sup_{x\in\rho\mathbb{B}} \big(d(x,S') - d(x,S)\big)^+ \quad \text{et} \quad \widehat{\operatorname{exc}}_\rho(S,S') := \sup_{x\in S\cap\rho\mathbb{B}} d(x,S').$$

Dans cet exposé, nous ferons dans un premier temps un tour d'horizon des résultats d'existence de solutions pour des processus de Moreau sous l'hypothèse d'un contrôle tronqué ([3, 2, 7]). Nous présenterons ensuite nos nouveaux résultats provenant de [5, 6].

Références

[1] S. Adly, *A variational Approach to Nonsmooth Dynamics*, SpringerBriefs in Mathematics, 2018.

[2] S. Adly and B.K. Le, *Unbounded second-order state-dependent Moreau's sweeping Processes in Hilbert spaces*, J. Optim. Theory Appl. 169 (2016), 407-423.

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[5] F. Nacry, *Truncated nonconvex state-dependent sweeping process : implicit and semi-implicit adapted Moreau's catching-up algorithms*, J. Fixed Point Theory Appl. 20 (2018)

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[7] L. Thibault, *Moreau sweeping process with bounded truncated retraction*, J. Convex Anal. 23 (2016), 1051-1098.

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A convergence result for a vibro-impact problems with a nonconvex moving set of constraints

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Nang Thieu Nguyen
Université de Limoges, France

Mots-clefs : vibro-impact problem, prox-regular sets, position-based time-stepping algorithm, normal cone

We are interested in the position-based time-stepping algorithm for vibro-impact problems with a nonconvex moving set of constraints and prove its convergence to a solution of a second order differential inclusion. Our results complement and extend some existing results in the literature ([2], [3], [4]).

Références

[1] S. Adly, F. Nacry and L. Thibault, *Preservation of prox-regularity of sets with applications to constrained optimization*, SIAM J. Optim. 26 (2016), no. 1, 448–473 .

[2] F. Bernicot and A. Lefebvre-Lepot, *Existence results for nonsmooth second-order differential inclusions, convergence result for a numerical scheme and application to the modeling of inelastic collisions*, Confluentes Math. 2 (2010), no. 4, 445–471.

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Global Probability of Collision for Space Encounters : Problem modeling via occupation measures

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Mioara Joldes
LAAS-CNRS

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Mots-clefs : Probability of collision, occupation measure, Liouville equation, linear programming on measures, Lasserre hierarchie.

Since the collision between the Russian satellite COSMOS 1934 and one debris of COSMOS 926 in December 1991, no less than eight orbital collisions have been reported between operational satellites, or between satellites and debris. Collision risk is particularly high in low orbits and the different space agencies (CNES, ESA, NASA) and the operators of the field (Airbus Defense and Space, GMV) have established alert procedures to assess the risks of collision for controlled satellites, and to authorize avoidance maneuvers if the predicted risk exceeds some tolerance threshold. At the origin of any procedure of collision avoidance between two objects controlled or not in orbit, is the information of conjunction between the two objects. Since 2009, a Conjunction Message is sent by the Joint Space Operations Center (JSpOC) to all spacecraft owners and operators, concerning approximately 15000 objects listed in the Two-Line Elements catalog provided by USSTRATCOM (US Strategic Command). The information provided by the JSpOC consists of a Conjunction Assessment Report (CAR) containing few information : the Time of Closest Approach (TCA), the miss distance between the two objects, statistical and geometrical information on the position and the velocities of each object. These messages are sent only three days before the date of the encounter. To obtain more accurate information on the possible encounter, it is necessary to subscribe to a service which will in return provide a Conjunction Summary Report (CSM) from which is extracted the information needed to calculate the risk of collision between both objects. This collision risk assessment evaluates the risk for individual encounters.
This talk is devoted to the collision risk assessment for space encounters between one operational spacecraft and one space debris. Let $[0,T]$ be the time interval of the encounter. Consider the dynamics of the 2 objects :

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & f(t,x(t)), \quad t \in [0,T], \\ x(0) & = & x_0. \end{array} \right. \tag{2}$$

where the state of reach orbiting object is described by their position and velocity in a reference frame $\mathcal{R}$ gathered in the global state vector : $x = (r_p, \dot{r}_p, r_s, \dot{r}_s) \in \mathbb{R}^{n=2\times 6}$. These equations include the Newtonian gravitational central field and possible orbital perturbations (non spherical Earth, atmospheric drag, e.g.). Whatever model is adopted, it is assumed that, for given initial conditions x_0, the solutions of the system (2), denoted by $x(t|x_0)$, exist and are unique.
The initial conditions $x_0 \in \mathbb{R}^n$ in positions and velocities are subject to uncertainties, usually represented as random vectors determined by their probability density functions. The orbital state uncertainty of each object involved in the conjunction is often assumed distributed according to a Gaussian distribution law : $x_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$, whose mean μ_0 and covariance matrix Σ_0 are given in the alert report send by the JSpOC. Classically the objects are assumed to be spherical : this assumption enables to ignore the orientation of the objects, and to model conservatively the secondary object whose geometry is often poorly known.
Let us now define the notion of a collision. Roughly speaking, a collision occurs when a trajectory enters a specified *forbidden region*. The domain of collision $\mathcal{D}_c([0,T])$ over the time interval $[0,T]$ is then defined as the set of initial conditions leading to a collision during $[0,T]$, namely :

$$\mathcal{D}_c([0,T]) = \{x_0 \in \mathbb{R}^n \mid \exists t \in [0,T],\ x(t|x_0) \in \mathcal{X}_R\},$$

where : $\mathcal{X}_R = \{x \in \mathbb{R}^{12} \mid \|r_p - r_s\|_2^2 \leq R^2\}$ denotes the forbidden region where the relative distance between the two objects is less than a certain given radius threshold $R > 0$. We can also introduce the set $\mathcal{X}_0$ of *safe* initial conditions i.e. not leading to a collision over $[0,T]$:

$$\mathcal{X}_0 = \{x_0 \in \mathbb{R}^{12} \mid \forall t \in [0,T],\ x(t|x_0) \in \mathcal{X}_R^c\}.$$

A first formulation of the collision risk assessment problem consists in computing the probability that no collision occurs i.e. the probability that initial conditions are safe, that is :

$$\mathcal{P}_{nc} = \mathbb{P}(x_0 \in \mathcal{X}_0) = \rho_0(\mathcal{X}_0), \tag{3}$$

The collision probability is then given by : $\mathcal{P}_c = 1 - \mathcal{P}_{nc}$. The analytical calculation of this probability is a very difficult problem : the first issue is to determine the domain of integration, which strongly depends on the chosen model for the dynamics when propagating the distribution of probability of the initial state. In addition, the integration of the density of probability on this set may be very complex, even for a Gaussian distribution. The most general methods to accurately compute the global collision probability, without any additional assumption, are based on Monte-Carlo simulations [2]. But these simulations can be dramatically time-consuming which makes these methods unsuitable for detecting low probability events especially in high dimension.
In the specific context of short-term encounters (high relative velocities, rectilinear trajectories), several techniques for calculating the probability of collision have been developed [4, 8, 1, 3, 9]. Unfortunately, these approaches are relatively limited because of their characterization for particular relative trajectories and are only imperfectly generalized to other context as satellite flying formation or proximity operations.
In this talk we propose a new modeling of the collision probability via occupation measure. First we will show that the problem of computing (3) can be reformulated as an infinite-dimensional linear programming (LP) problem in the cone of nonnegative measures whose optimal value is the expected non-collision probability. The main tool is the Liouville equation, which appears in classical mechanics and describes the time evolution of a measure transported by the flow of a nonlinear dynamical system [6, 5]. Our main contribution is to propose the first exact

and rigorous mathematical modeling for the general case of collision probability computation, without any simplifying assumption. Secondly we will focus on a practical way of solving LP on measures via the manipulation of their moments which give rise to *so-called* hierarchies of SDP relaxations [7], which account to solving a sequence of SDP optimization problems of increasing size and whose objective converges to the solution of the initial LP on measures. Some numerical results will be presented to illustrate the strenght and the limits of our approach in practice.

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A complementarity-based approach to cardinality-constrained optimization

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Sparse optimization problems and optimization problems with cardinality constraints have many applications such as portfolio optimization, subset selection, compressed sensing or learning. In the past, solution approaches have often focused on convex substitutes of the respective problems, e.g. using the l1-norm to induce sparsity. However, recently non convex formulations have gained popularity. In this talk, we give an introduction to a complementarity-based solution approach for cardinality-constrained optimization problems.

Principle de Morozov via la Dualité de Lagrange

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Mots-clefs : Ill-posed problems, Morozov Principle, Lagrange duality

Dans le domaine des problèmes inverses et des méthodes de régularisation, la sélection du paramètre de régularisation "smoothing parameter" est une étape cruciale pour une bonne approximation de la solution du problème donné. Dans cet exposé,nous présentons une méthode assez pratique et rapide pour le calcul du paramètre de régularisation suivant le célèbre principe de Morozov. Les résultats de simulations numériques de cette méthode sont aussi presentés, L'exemple considéré étant un problème inverse bien connu en statistiques : La régression instrumentale non-paramétrique.

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Contrôle périodique : généralités et applications

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Mots-clefs : Contrôle optimal, Principe de Maximum de Pontryagin, Solutions Périodiques, Over-yielding, Commande hybride.

Dans cet exposé on présente l'étude d'un problème de contrôle optimal sous contrainte intégrale sur la commande ([1]). On considère un système dynamique scalaire, linéaire en u qui est la variable de commande. On montre d'abord l'existence des solutions périodiques non-constantes, associées à une commande qui vérifie une contrainte intégrale. On présente après les conditions (globales) permettant d'améliorer un coût moyen (par rapport à une commande constante), avec une commande périodique non constante qui a une moyenne fixée sur la période (égale à celle de la commande constante). Ensuite, on applique le Principe de Maximum de Pontryagin afin de trouver la solution optimale périodique ([2]). On donne aussi quelques exemples d'application en dynamique de populations : modèle du chémostat mono-spécifique et modèle d'exploitation bioéconomique de ressources renouvelabes ([3], [4], [5]).
Pour le modèle du chémostat à deux espèces, on propose une commande hybride, qui permet à une espèce 1 de dépasser par alternance un seuil, lorsqu'une autre espèce 2 (non désirable) est présente dans le milieu de culture ([6], [7]). Ainsi, on observe que le temps passé par l'espèce 1 au dessus du seuil est de mesure infinie. Cette commande périodique génère également des solutions asymptotiquement périodiques.

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Selfcontracted curves, applications and extensions

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Mots-clefs : Gradient flows, self-contracted curves, rectifiability.

A class of curves encompassing all orbits generated by a gradient flow of a quasiconvex potential has been defined in [2]. These curves are called self-contracted curves and enjoy the following simple metric definition : Let $\gamma : I \subset \mathbb{R} \to (X, d)$ be a curve where I is an interval and (X, d) is a metric space. The curve γ is called self-contracted if for every $t_1 < t_2 < t_3$ in I, the following inequality holds :

$$d(\gamma(t_1), \gamma(t_3)) \geq d(\gamma(t_2), \gamma(t_3)).$$

In [3], when the ambient space is the Euclidean space $\mathbb{R}^d$, an upper bound for the length of the aforementioned curves is given depending only on the dimension d and the diameter of the convex hull of the image of the curve. This result has important consequences in the study of convergence of the proximal algorithm method or even in dynamics given by a convex foliation. Recently, different directions have appeared in the study of self-contractedness. One of them is extending the notion of the aforementioned class of curves, as it has been done in [4] (λ-curves and λ-eels). Many questions arise out of these generalizations. In this talk we shall present the main idea of [1] and we show how this idea can be used for the study of rectifiability for λ-curves, when $\lambda < 1/d$. We shall also present some results for λ-eels.

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Some new developments on the Campanato nearness condition

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Mots-clefs : Elliptic PDEs, Birkoff-James orthogonality, Campanato's nearness.

I propose to survey the theory of nearness between operators acting on normed spaces and developed by S. Campanato at the end of the eighties in a series of papers (see e.g. [1]). The aim of S. Campanato was to study existence and regularity results for some differential elliptic equations. Given X a set with at least two elements, and $(Y, \|\cdot\|)$ a real normed space, he said that the function $a : X \to Y$ is *near* the function $b : X \to Y$ if the inequality

$$\| (b(x_2) - \alpha\, a(x_2)) - (b(x_1) - \alpha\, a(x_1)) \| \leq \kappa \, \|b(x_2) - b(x_1)\| \qquad \forall x_1,\, x_2 \in X \tag{4}$$

holds for some positive constant α, and some real number κ such that $0 < \kappa < 1$.
Obviously nearness is a reflexive relation. The first part of the talk adresses the natural question of the **symmetry** of the nearness relation, as developed recently in [2]. We observe that when (Y is an **inner product space** and a is near b for the constants α and κ, then b is near a, but for the different constants $\frac{1-\kappa^2}{\alpha}$ and κ. When the dimension of Y is greater or equal to three, then the three following properties are equivalent : Y is an inner product space, the Birkhoff-James orthogonality is symmetric, and the Campagnato nearness is symmetric.
In a second part of the talk, I will propose an extension of the nearness property to multifunctions, as developed in [3], and investigate which properties of set-valued mappings are preserved by nearness.

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Local analysis of a regularized primal-dual algorithm for nonlinear programming without constraint qualification

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Mots-clefs : augmented Lagrangian method, interior point method, primal-dual algorithm, constraint qualification, quadratic/superlinear convergence, degenerate problem, regularization

In nonlinear optimization, the lack of the Mangasarian-Fromovitz Constraint Qualification (MFCQ) may lead to numerical difficulties and in particular to slow down the convergence of an optimization algorithm. In this talk, we analyze the local behavior of an algorithm based on a mixed logarithmic barrier-augmented Lagrangian method [1, 2] for solving a nonlinear optimization problem. This work has been motivated by the good efficiency and robustness of this algorithm, even in the degenerate case in which the MFCQ does not hold. Furthermore, we detail different updating rules of the parameters of the algorithm to obtain a rapid (superlinear or quadratic) rate of convergence of the sequence of iterates. The local convergence analysis is done by using a stability theorem of Hager and Gowda [3], as well as a boundedness property of the inverse of the regularized Jacobian matrix used in the primal-dual method. Numerical results on degenerate problems are also presented.

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