Constrained Derivative Free Optimization for Reservoir Characterization

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Summary

- Context and objectives of the reservoir characterization
- Derivative free optimization methods
- SQA method (Sequential Quadratic Approximation)
- Introducing constraints
- Results
  - on benchmark test cases
  - on a reservoir engineering application
- Conclusions and outlook

Reservoir characterization

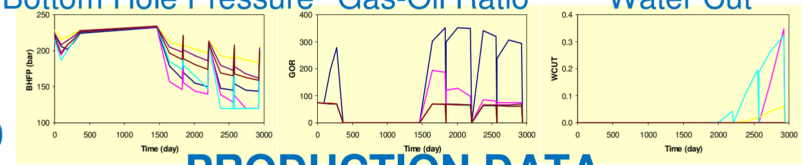
History matching from production data and 4D seismic data

for characterization of dynamic behavior of reservoir during the production of a field

- **Petrophysical parameters:**
  Porosity and permeability
  Fault properties
- **Well parameters: Skin, PI ...**

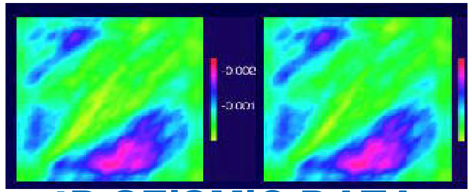
FORWARD → **problem** ← **INVERSE**

Bottom Hole Pressure Gas-Oil Ratio Water Cut



PRODUCTION DATA

Time 1 Time 2



4D SEISMIC DATA : impedance maps

Forward problem : fluid flow simulation in reservoir
petro-elastic modelling

Reservoir characterization

Characteristics of the optimization problem

$$\min_{m} f(m) := \|d_P(m) - d_P^{obs}\|_{C_P}^2 + \|d_S(m) - d_S^{obs}\|_{C_S}^2$$

- nonlinear least-square problem
- data space: up to 1.000.000 measurements
- parameter space: ~10 up to 100 (various types)
- gradient unavailable
- simulation expensive in computation time (1mn - hours)

Derivative free optimization methods

- Classical methods using gradients approached by finite differences

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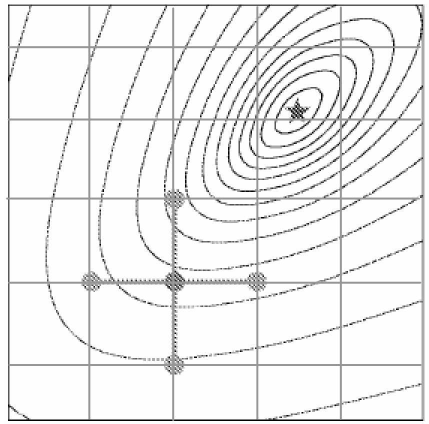
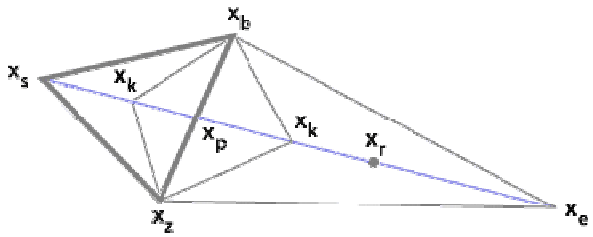
Derivative free optimization methods

- Classical methods using gradients approached by finite differences



- Pattern Search

- Nelder Mead Simplex



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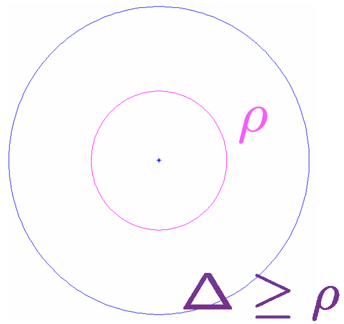
- Genetic algorithm (global): CMA-ES
- Optimization based on surrogate models
  - global model: (ex kriging + Expected Improvement criterion – D Jones)
  - local quadratic model in a trust region

SQA Method (Sequential Quadratic Approximation)

M.J.D. Powell, 2004, The NEWUOA software for unconstrained optimization without derivatives

1. Determine the first quadratic model Q interpolating the objective function in $n+2 \leq m \leq (n+1)(n+2)/2$ points in a trust region $\Delta = \rho$
2. At a given iteration

   A. Minimization of Q and update of Δ

   - $\min\limits_{\|d\| \leq \Delta} Q(x_{opt} + d)$
   - If $\|d\| < \dfrac{\rho}{2} \longrightarrow$ 2.B
   - Otherwise we use the new evaluation $f(x_{opt} + d)$ to update
     - the quadratic model Q
     - the radius of the trust region Δ according to the predictivity of the quadratic model R
   - If $R > 0.1 \longrightarrow$ 2.A

   B. Check the validity of Q in the trust region

   - not valid: add a point $\max\limits_{\|d\| \leq \Delta} |l_t(x_{opt} + d)|$, $f(x_{opt} + d)$ is evaluated
   - valid: If $\|d\| > \rho \longrightarrow$ 2.A, otherwise 3
3. Reduction of ρ and $\longrightarrow$ 2 or STOP

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Constraints in SQA

Derivatives of constraints are given

Taking into account constraints consists of 3 steps

1. The choice of initial points in the admissible domain defined by the constraints
2. The minimization of the model under constraints in the trust region

$$\min_{\|d\|\leq\Delta} Q(x_{opt}+d) \quad s.t. \begin{cases} C(x) \leq 0, \\ E(x) = 0. \end{cases}$$

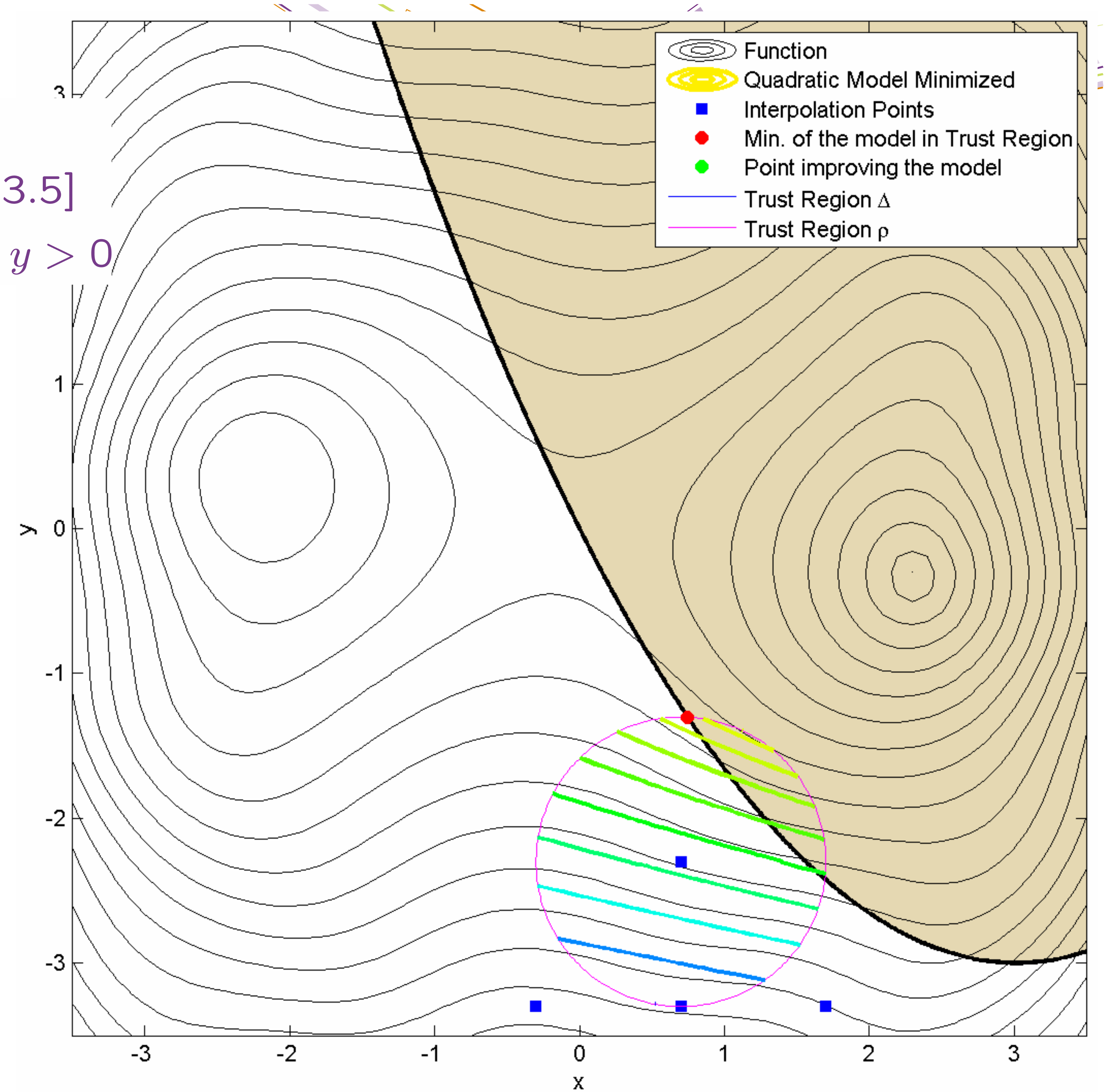
3. The improvement of the model under inequality constraints in the reduced trust region

$$\max_{\|d\|\leq\Delta} |l_t(x_{opt}+d)| \quad s.t.\ C(x) \leq 0.$$

calculated with SQPAL, a Sequential Quadratic Programming Approach
Sinoquet D. et Delbos F., 2007

$$\min_{x,y\in\mathcal{R}} f(x,y)$$

$$\text{s.t.} \begin{cases} x,y \in [-3.5, 3.5] \\ \frac{(x-3)^2}{3} - 3 - y > 0 \end{cases}$$



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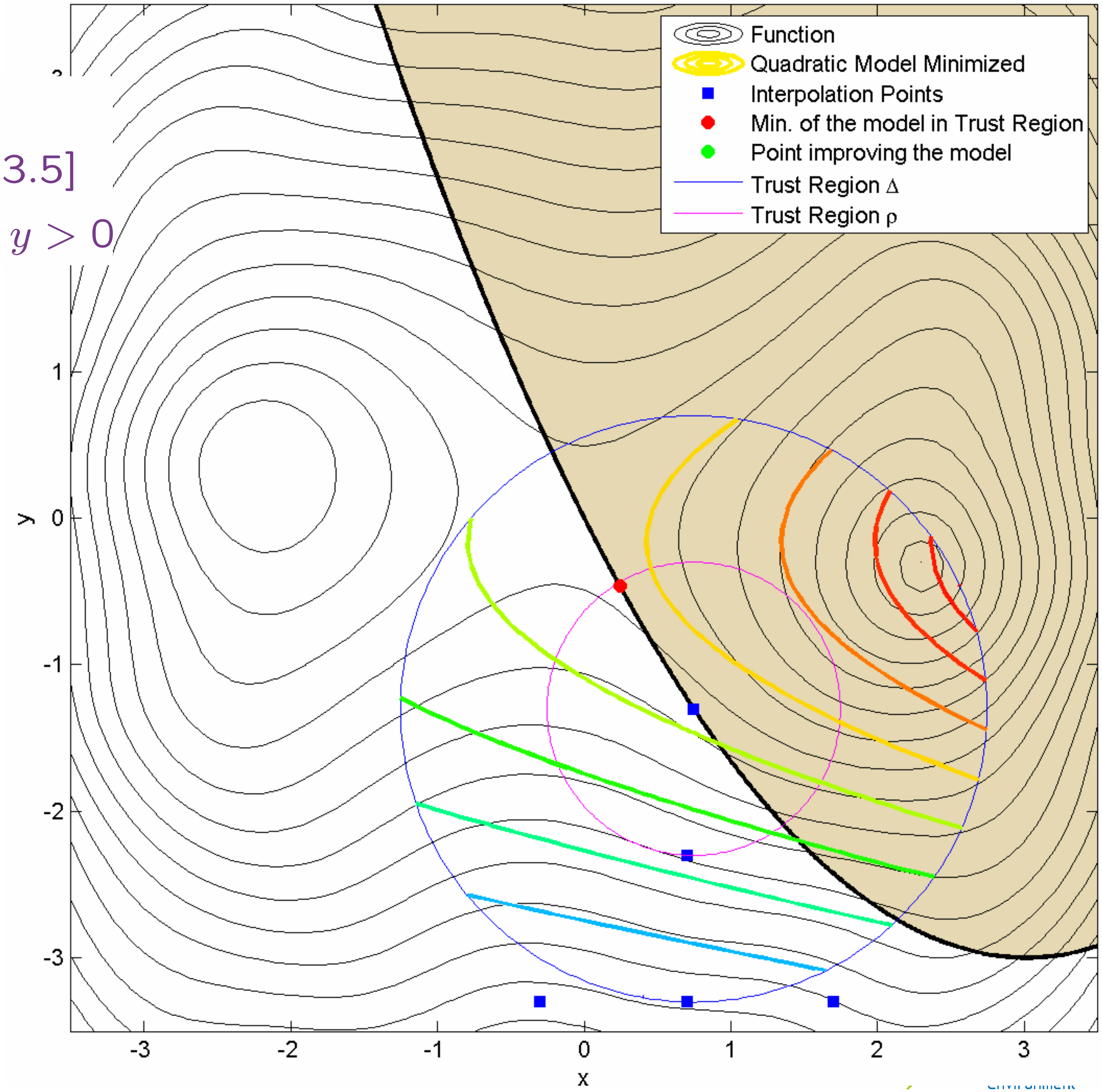

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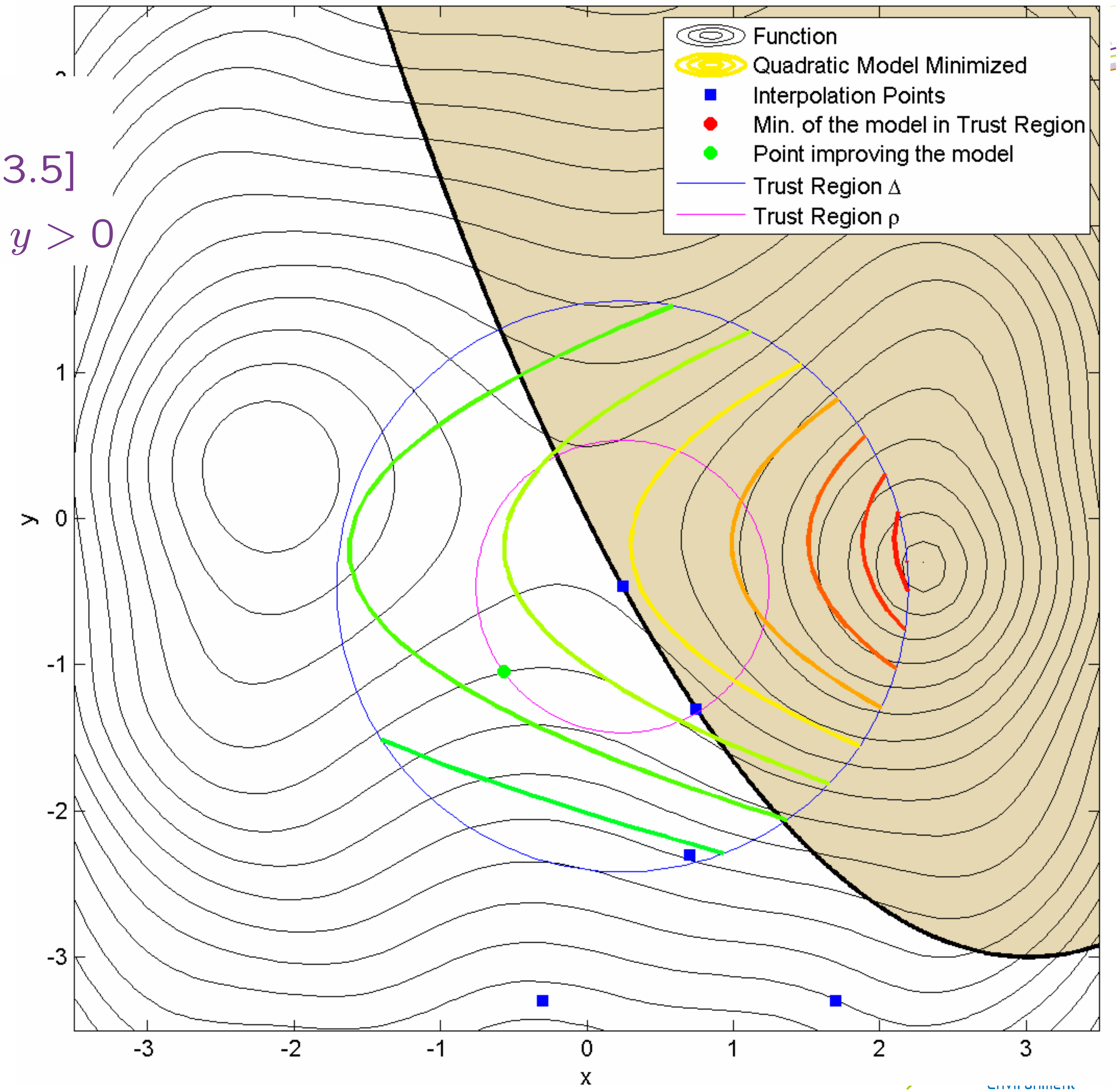

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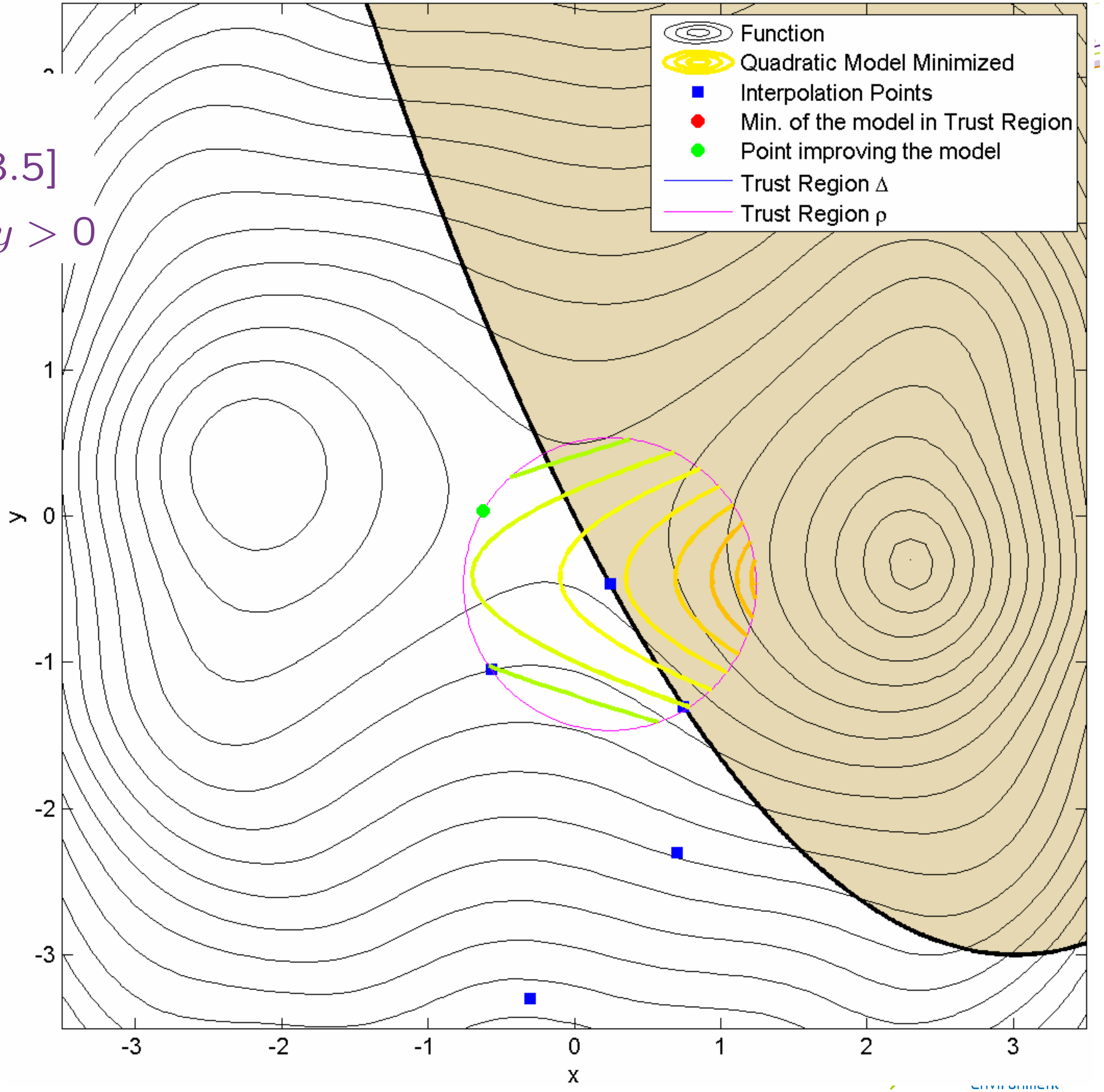

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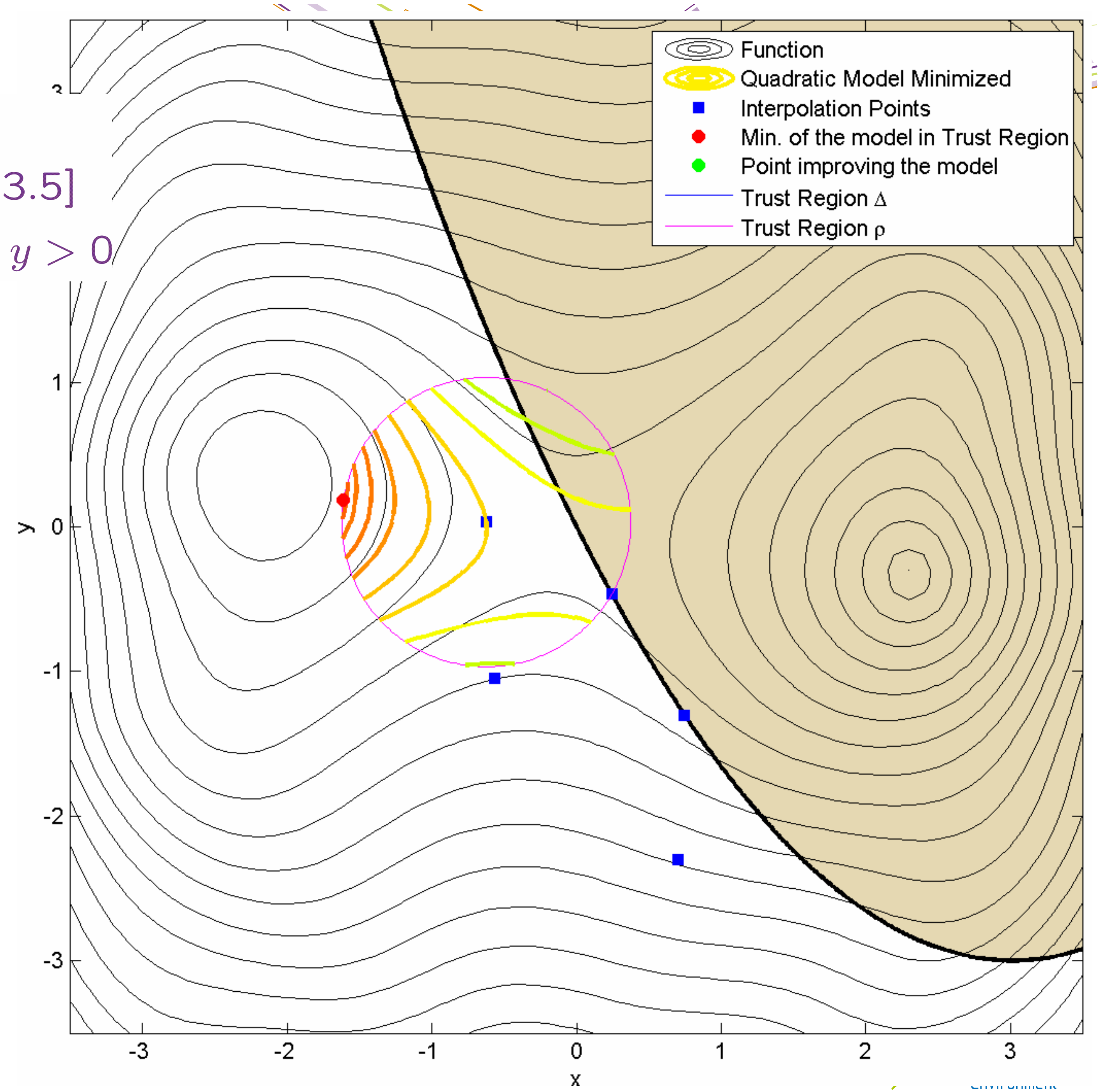


Results on More & Wild benchmark

Moré J.J. & Wild S.M., 2007,
Benchmarking Derivative-Free Optimization Algorithms

- 29 Test cases constructed from the benchmark CUTEr

$$f(x) = \sum_{k=1}^{m} f_k(x)^2$$

<u>under bound constraints</u>

- Dimension: 2 to 7 parameters

- Stopping Criterion: number of function evaluations

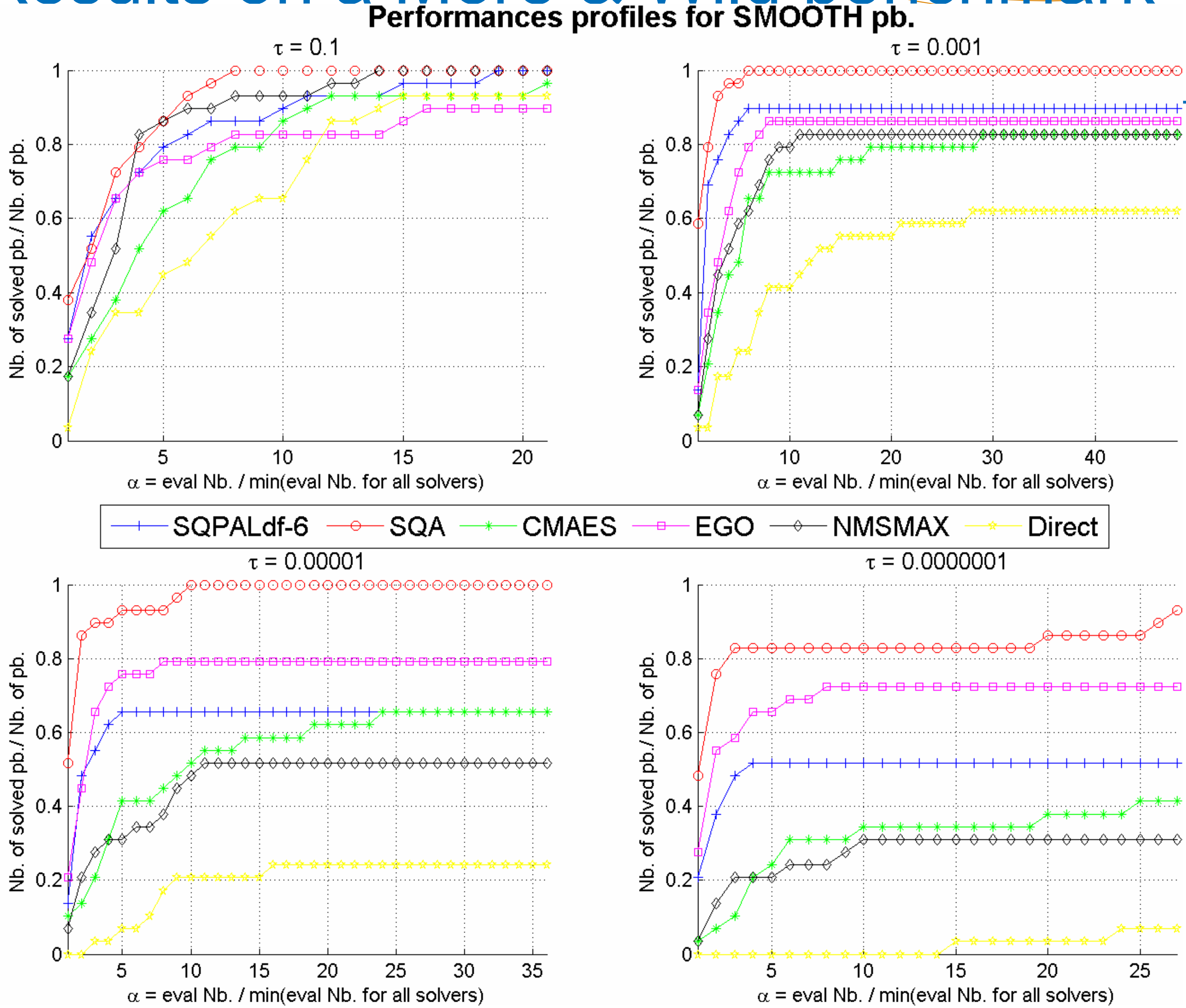
- Accuracy measured by $\tau = \dfrac{f(x_{n_{eval}}) - f_L}{f(x_0) - f_L}$

Results on More & Wild benchmark

Comparison between

SQA	Local quadratic model in a trust region
SQPAL	SQP method using gradient approached by finite differences
EGO	kriging with Expected Improvement
CMAES	Genetic Algorithm
NMSMAX	Nelder Mead Simplex
Direct	Pattern Search

Results on a More & Wild benchmark



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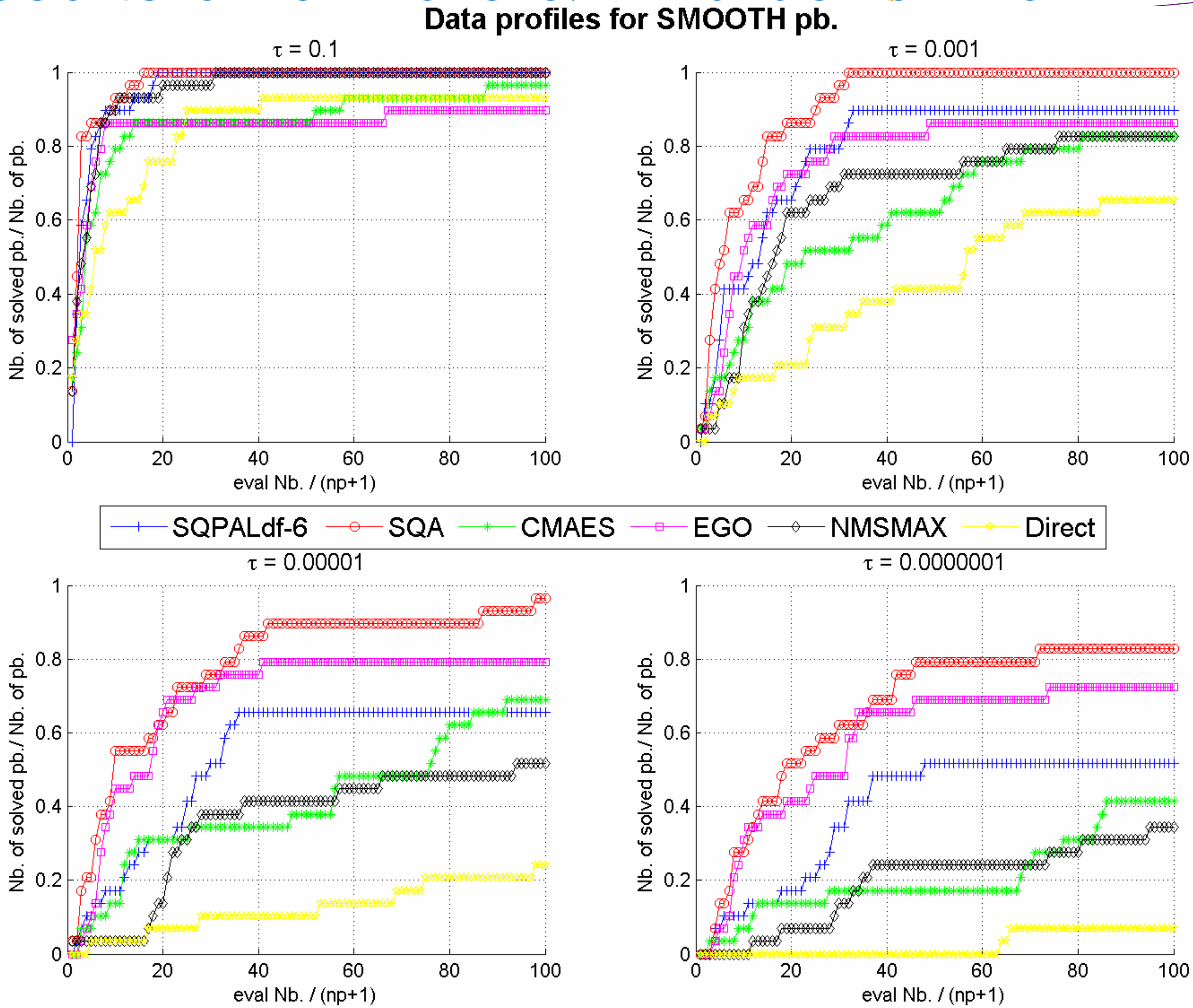
Results on a More & Wild benchmark



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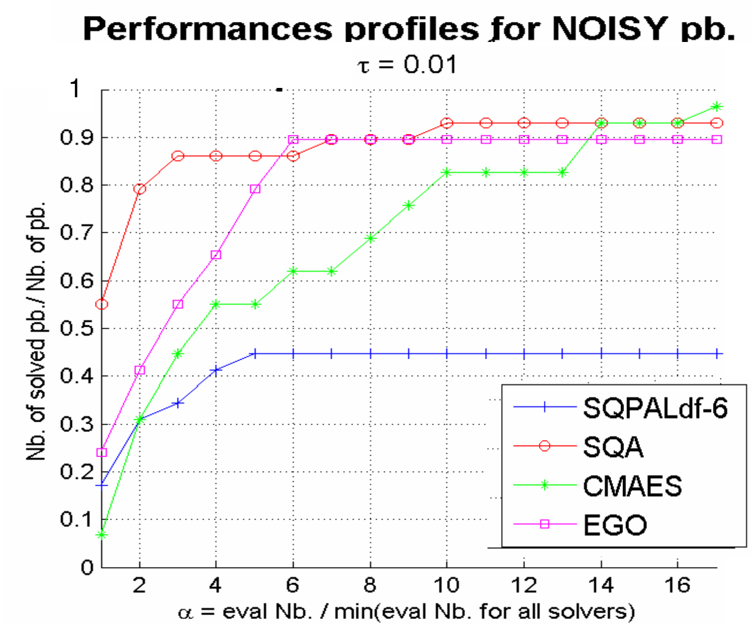
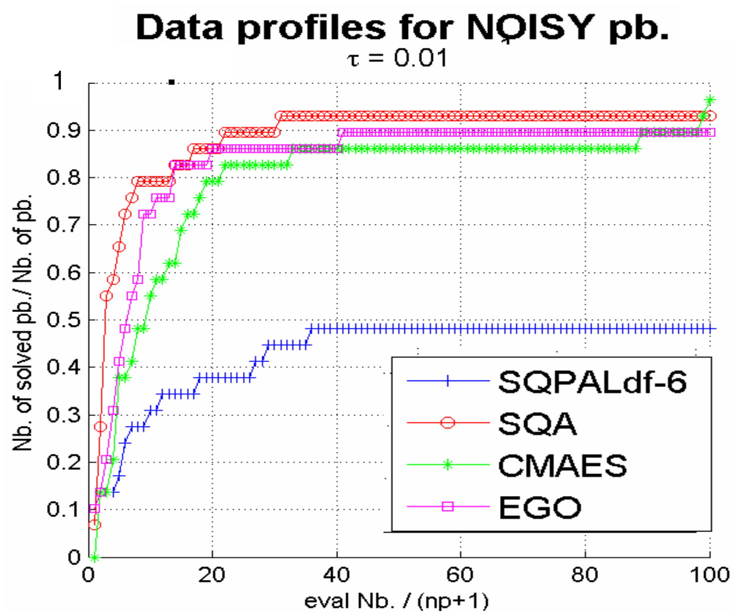
Results on a More & Wild benchmark

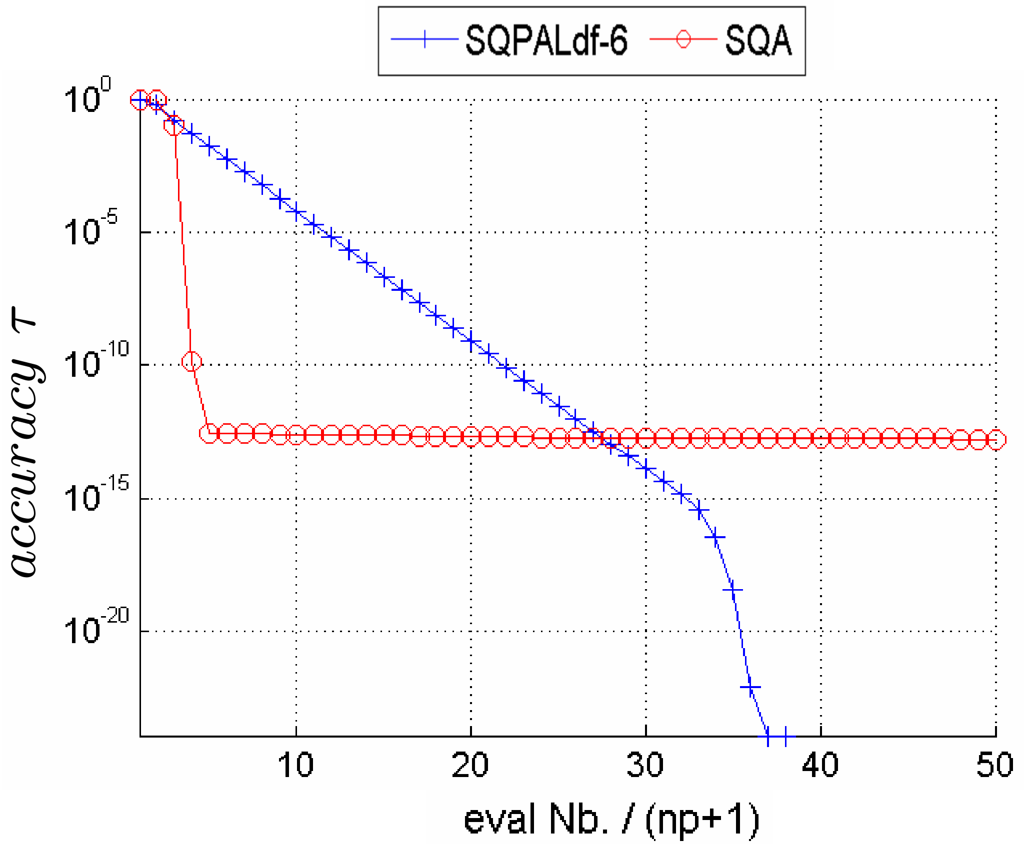
$\text{noise} = 10^{-2}$





Example in high dimension

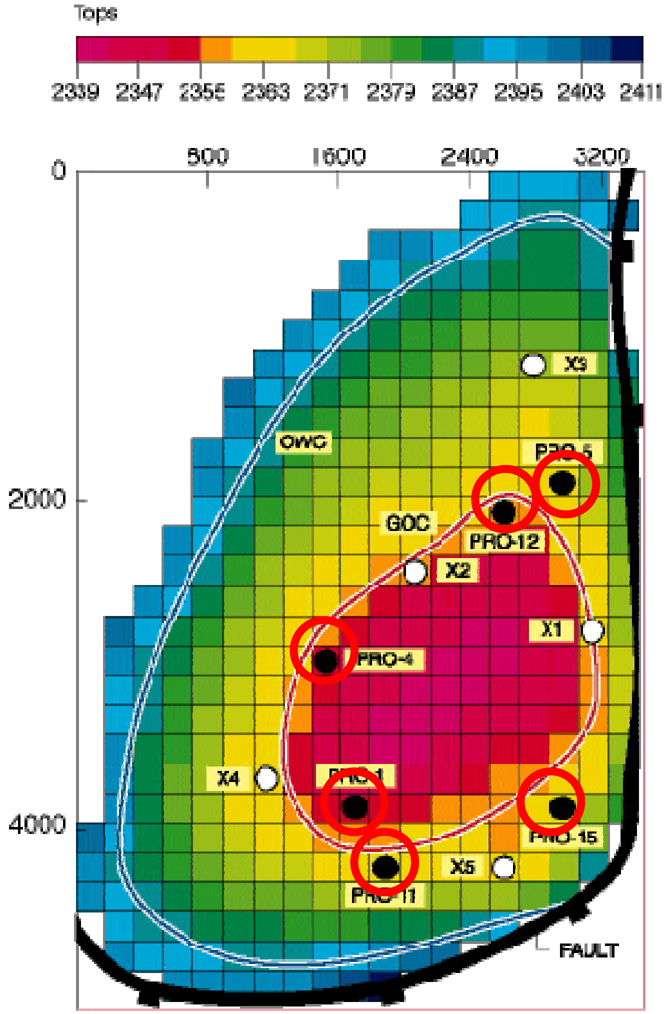
dimension $n = 100$



vardim example

$$f(x) = \sum_{i=1}^{n} (x_i - 1)^2 + \left(\sum_{i=1}^{n} i x_i - \frac{n(n+1)}{2}\right)^2 + \left(\sum_{i=1}^{n} i x_i - \frac{n(n+1)}{2}\right)^4$$

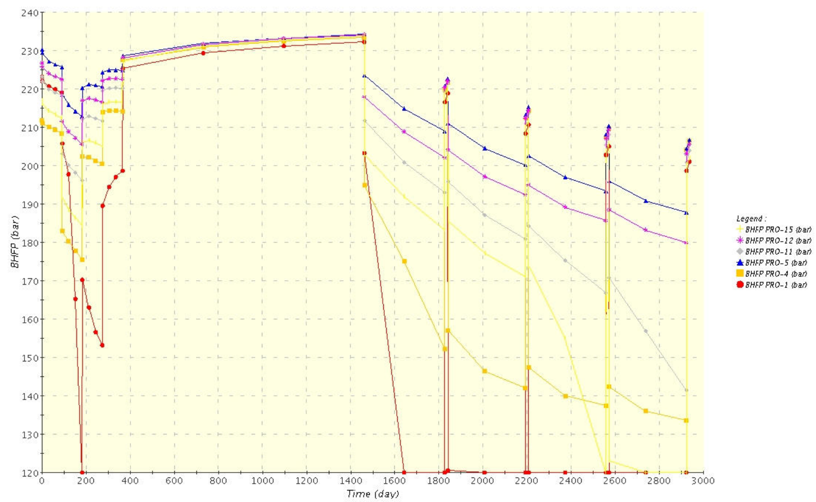
Results on reservoir application



PUNQS XY plane 1

Data: production data from 6 wells

778 measurements (3 per well at 41 ≠ days)

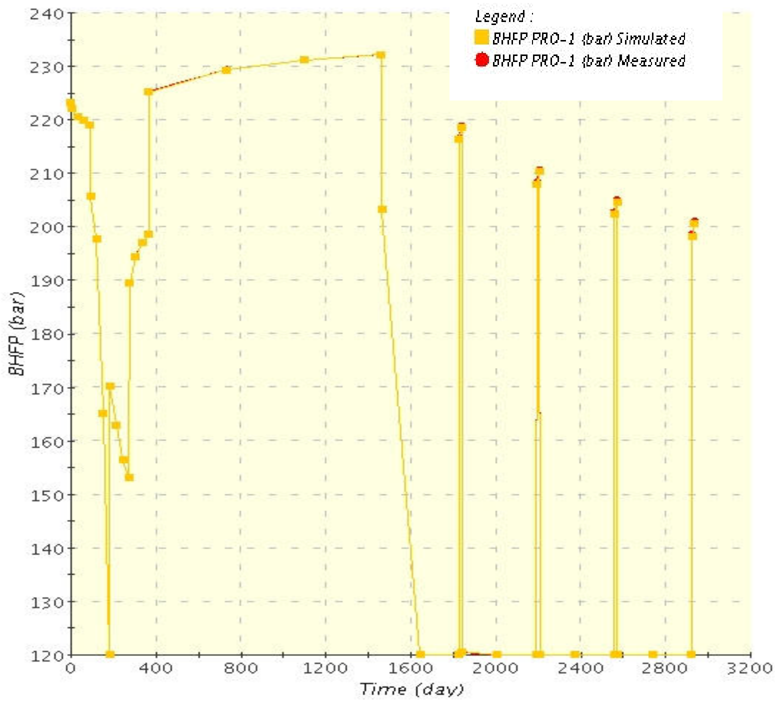


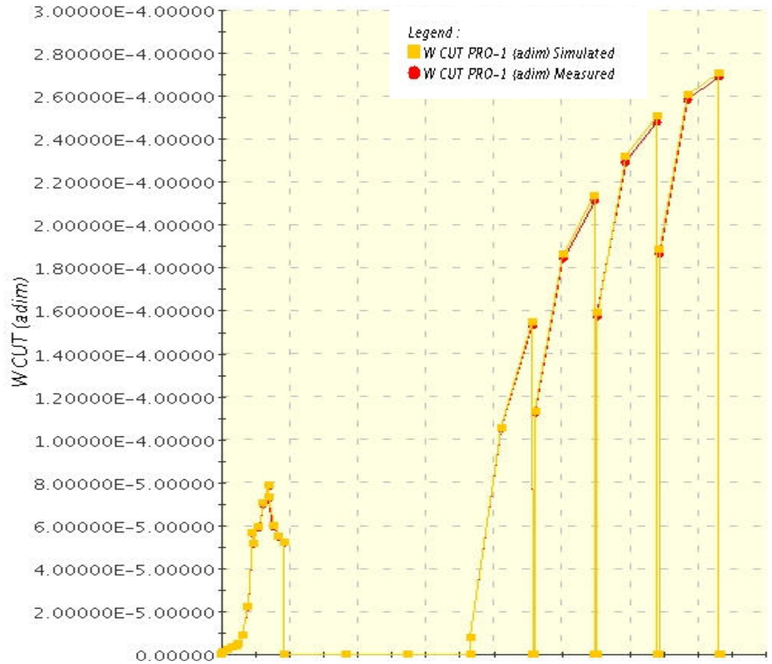
Parameters : 10 or 40

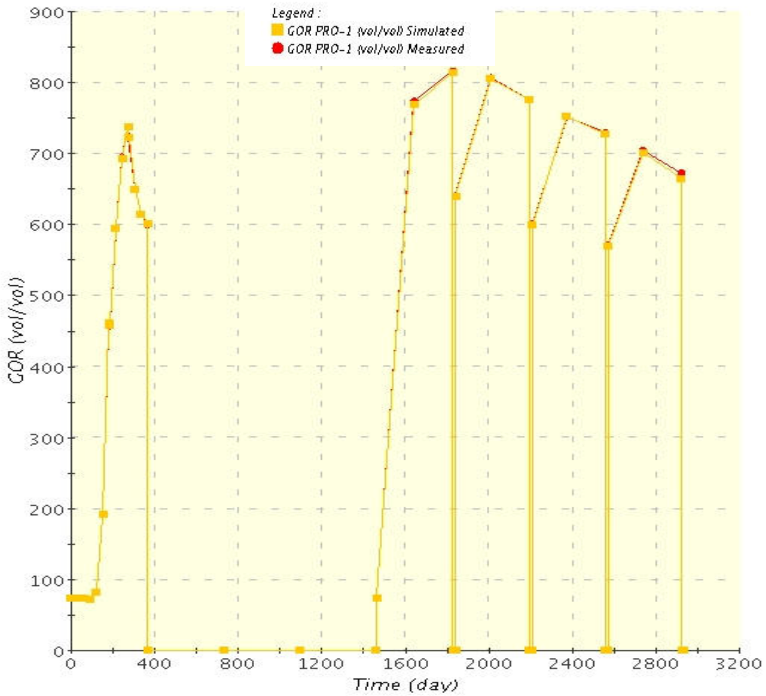
- geostatistical parameters (10)
- rocks properties (30)

Results on reservoir application

10 parameters







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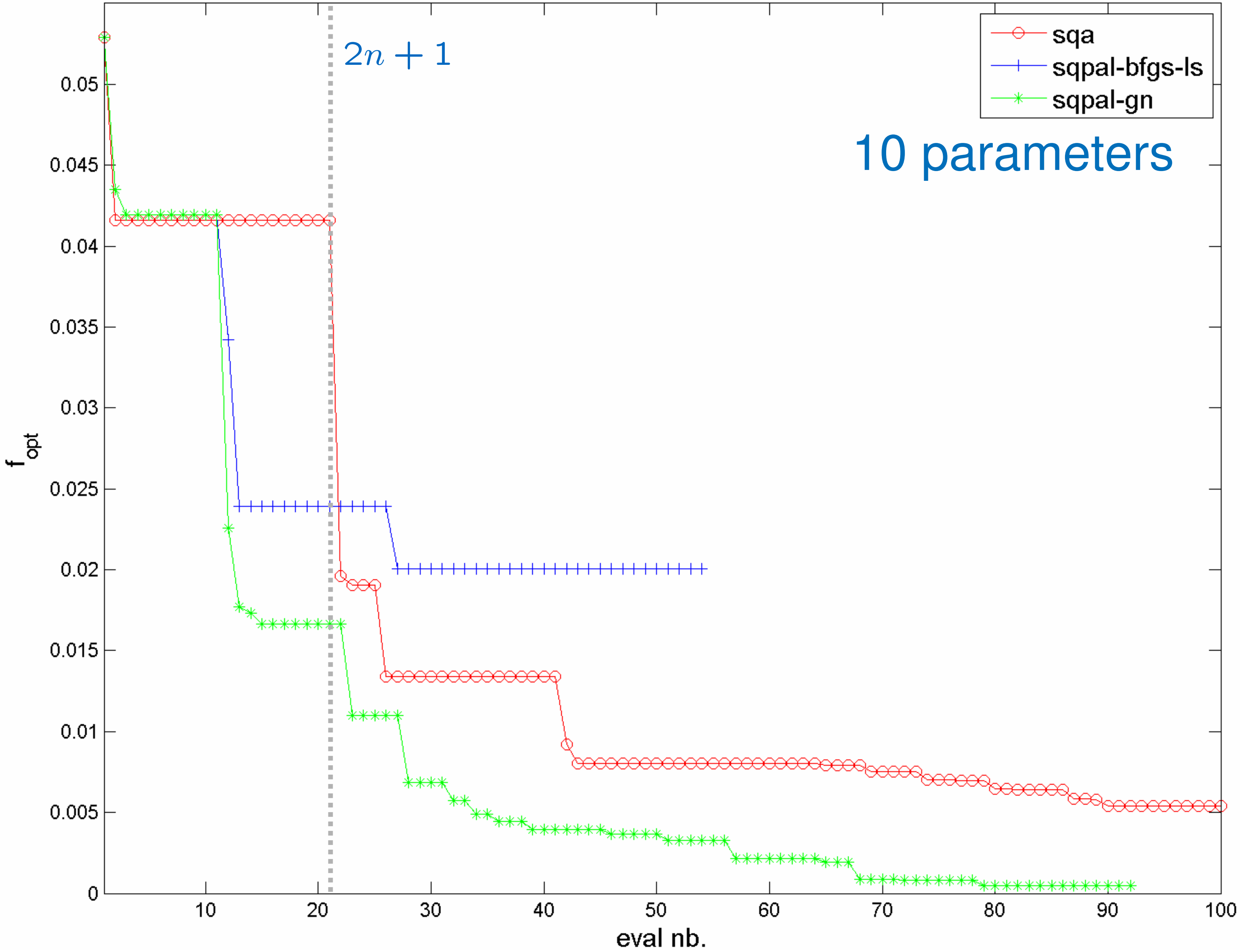

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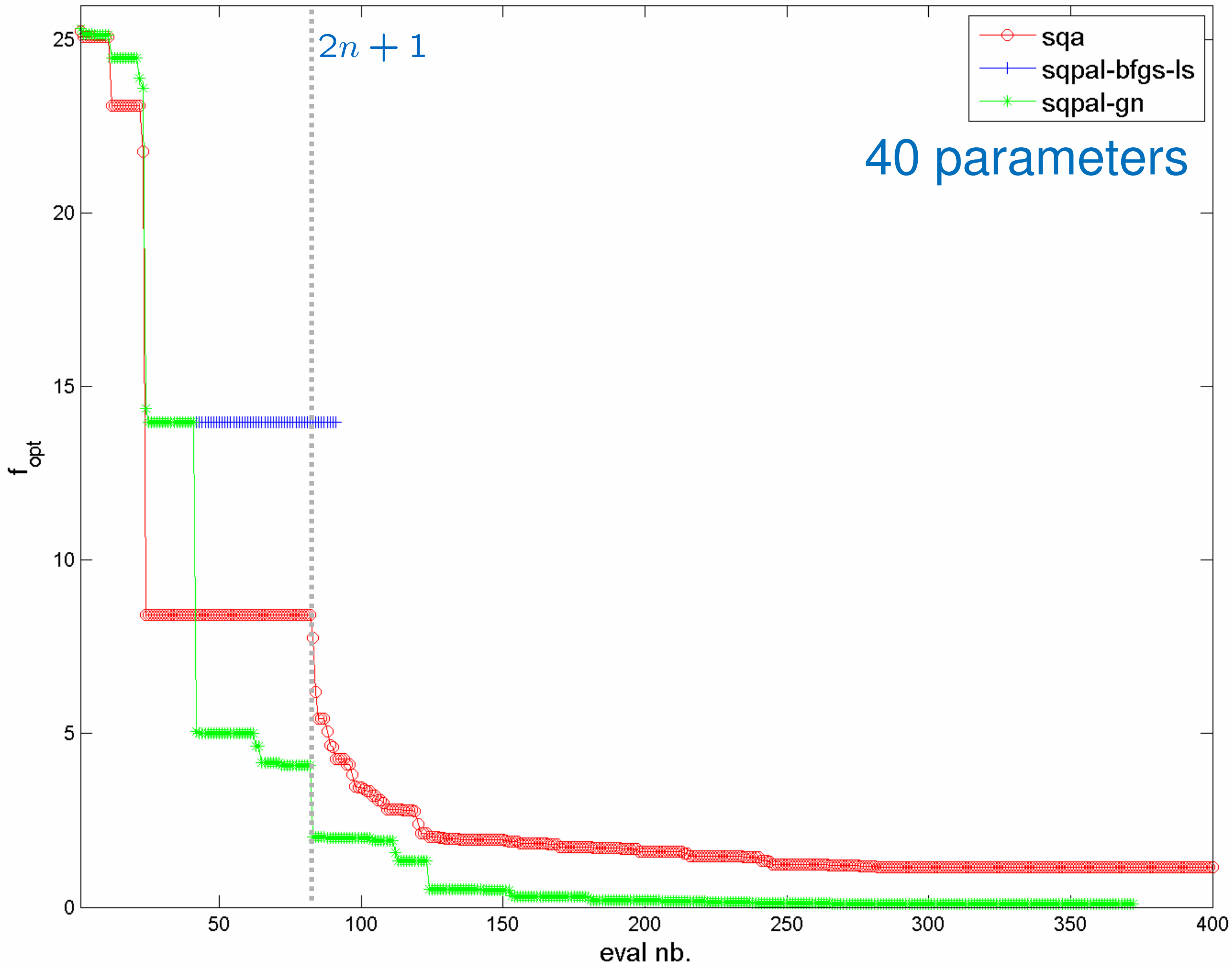

Conclusions and outlook

- SQA
  - Very efficient method for optimization without derivatives
    - SQA better than EGO and SQPAL (Finite differences) on test cases
    - SQA can deal examples of more than 100 parameters (≠ EGO)
    - extension of SQA to nonlinear constraints (with derivatives)
  - First results promising on the application in reservoir characterization.
    Next step : test with constraints
- Outlook
  - adapt SQA to take into account nonlinear constraints without derivatives available (for other applications: engine calibration)
  - adapt SQA for least square problems (inverse problems)

    Zhang, Conn, Scheinberg, 2009, A Derivative Free Algorithm for the least-square minimization