Decomposition of high dimensional aggregative control problems
Application to energy management

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Joint work with

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14 janvier 2021

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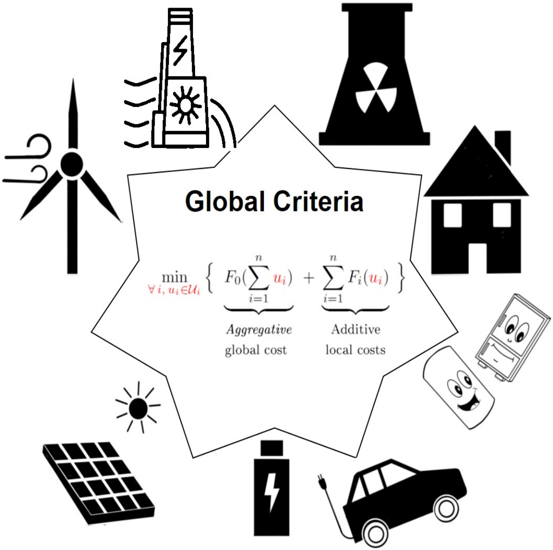
Outline

Controlling a large number of electric flexibilities



Actors

- **Central actor**: aggregator, TSO, producer/provider, . . .
- **Local flexibilities**: generation, storage or flexible consumption (AC, heating, fridge, water-heater, etc.), . . .

Goal Optimize the **Global Criteria** while

- satisfying flexibilities technical constraints
- preserving Quality of Service of devices
- preserving privacy of consumers (individual consumption or constraints)
- limiting computing time and communications

Aggregative global cost + additive local costs

Problem Control the aggregate of generation/consumption provided by a population of flexibilities $i = 1, \cdots, n$ in order to

$$\min_{\forall\, i,\, u_i \in \mathcal{U}_i} \left\{ \underbrace{F_0(\sum_{i=1}^{n} u_i)}_{\substack{\textit{Aggregative} \\ \text{global cost}}} + \underbrace{\sum_{i=1}^{n} F_i(u_i)}_{\substack{\text{Additive} \\ \text{local costs}}} \right\}$$

***Aggregative* global cost**

- $F_0(\sum_{i=1}^{n} u_i) := d(r, \sum_{i=1}^{n} u_i)$ tracking a target profile $(r_t)_{t=1,\cdots,T}$,

- $F_0(\sum_{i=1}^{n} u_i) := \Big\{$ Balancing cost (generation +market position) to satisfy the inflexible demand and flexible demand $d + \sum_{i=1}^{n} u_i$ at lowest cost

Local features must be kept private

- local costs F_i
- local constraints: admissible sets, $\mathcal{U}_i$
- **Uncertainties** may impact local agents
- **Large number** of agents: $> 10^3, \cdots$ or even $> 10^6$

Outline

A high dimensional stochastic control problem (1/2)

Problem Control the consumption of a population of flexibilities $i = 1, \cdots, n$ in order to conciliate local costs with a global cost:

$$(P_1) \quad \begin{cases} \inf_{u \in \mathcal{U}} J(u) \\ J(u) := \mathbb{E}\left(F_0\big(\frac{1}{n}\sum_{i=1}^{n} u^i\big) + \frac{1}{n}\sum_{i=1}^{n} F_i(u^i, X^{i,u^i}) \right). \end{cases}$$

• **the state process $X^u := (X^{i,u^i})_{i\in\{1,\dots,n\}}$** models the state of a population of flexibilities evolving randomly according to the control process $u = (u^i)_{i\in\{1,\dots,n\}}$, $u^i = (u^i_t)_{t\in[0,T]}$

$$\begin{cases} dX_t^{i,u^i} &= \mu_i(t, u_t^i, X_t^{i,u^i})dt + \sigma_i(t, X_t^{i,u^i})dW_t^i, \quad \text{for } t \in [0,T], \\ X^i_{0,u^i} &= x_0^i \in \mathbb{R}. \end{cases} \tag{1}$$

where $W := (W^i)_{i\in\{1,\dots,n\}}$ stands for an n dim. Brownian Motion (BM).

Example $X^u := (X^{i,u^i})_{i\in\{1,\dots,n\}}$ may represent indoor-temperatures of a population of fridges with ON/OFF decisions $u^i_t \in \{0, 1\}$.

A high dimensional stochastic control problem (2/2)

Problem Control the consumption of a population of flexibilities

$$(P_1) \quad \begin{cases} \inf_{u \in \mathcal{U}} J(u) \\ J(u) := \mathbb{E}\left(F_0\left(\frac{1}{n}\sum_{i=1}^n u^i\right) + \frac{1}{n}\sum_{i=1}^n F_i(u^i, X^{i,u^i}) \right). \end{cases}$$

- **The set of admissible controls** $\mathcal{U}$ is composed of elements $u = (u^1, \ldots, u^n) \in \mathcal{U} = \mathcal{U}_1 \times \ldots \times \mathcal{U}_n$ s.t. for all $i \in \{1, \cdots, n\}$
  - u^i **is a progressively measurable** process w.r.t. the *large* filtration $(\mathcal{F}_t)_{0 \le t \le T}$ ($\neq$ $(\mathcal{F}^i_t)_{0 \le t \le T}$) where

$$\mathcal{F}_t := \sigma(W_s \,|\, 0 \le s \le t) \quad \text{while} \quad \mathcal{F}^i_t := \sigma(W^i_s \,|\, 0 \le s \le t)$$

  - u^i takes **bounded values** s.t.

$$u^i_t(\omega) \in [-M_i, M_i] \quad \text{for } t \in [0, T],\ 0 < M_i < \infty\ .$$

- **Example** in Dynamic Programming, costs have a specific form of the type

$$F_0(v) = \int_0^T f_0(t, v_t)dt\ , \quad \text{and} \quad F_i(u^i, X^{i,u^i}) = \int_0^T f_i(t, u^i_t, X^{i,u^i}_t)dt$$

Numerical issues / Existing approaches

- **A stochastic control problem with challenging numerical issues**
  - high dimensional state space (n *is big !*)
  - high dimensional noise (n *is big !*) but no common noise
  - continuous time $\Rightarrow$ large number of time steps after discretization
- **Existing approaches**
  - **Random trees** [HigleEtal13, RuszczynskiEtal03]
    uncertainties are approximated by a random tree, then decomposition methods are implemented [RockafellarEtal91, Salinger97].
    But **limited to problems with a small noise space and few time steps**.
  - **Dynamic Programming** [Bertsekas04]
    ▶ requires Markov property of the system and a **specific structure of costs functions**.
    ▶ **curse of (state space) dimensionality** limits classical approaches up to dimension 5.
    ▶ **Stochastic Dual Dynamic Programming (SDDP)** [PereiraEtal91]
    allows to consider states **up to dimension 30**, under convex assumptions.
  - **Dual Approximate Dynamic Programming (DADP)**
    [Strugarek06, Girardeau10, Alais13, Leclère14, CarpentierEtal18]
    consists of a price decomposition, where the **stochastic constraints are projected on subspaces** s.t. the Lagrangian multiplier is adapted for dynamic programming.
    But in general no error bound can be estimated a priori.

The present approach follows the same spirit as DADP but provides an a priori error bound ans relies on an original stochastic algorithm.

Outline

Three main steps

1. **Approximation of the original problem** $(P_1) \approx (P_2)$

$$(P_1) \quad \inf_{u\in\mathcal{U}} J(u) \quad \text{with} \quad J(u) := \mathbb{E}\left(F_0\Big(\frac{1}{n}\sum_{i=1}^n u^i\Big) + \frac{1}{n}\sum_{i=1}^n F_i(u^i, X^{i,u^i})\right)$$

by the auxiliary problem

$$(P_2) \quad \inf_{u\in\mathcal{U}} \tilde{J}(u) \quad \text{with} \quad \tilde{J}(u) := F_0\left(\frac{1}{n}\sum_{i=1}^n \mathbb{E}(u^i)\right) + \frac{1}{n}\mathbb{E}\left(\sum_{i=1}^n F_i(u^i, X^{i,u^i})\right)$$

2. **Lagrangian decomposition of** $(P_3) \Leftrightarrow (P_2)$

$$(P_3) \begin{cases} \min\limits_{u\in\mathcal{U}, v\in\mathcal{V}} \bar{J}(u,v), \quad \text{with} \quad \bar{J}(u,v) := F_0(v) + \dfrac{1}{n}\mathbb{E}\left(\displaystyle\sum_{i=1}^n F_i(u^i, X^{i,u^i})\right) \\ \text{s.t } \dfrac{1}{n}\displaystyle\sum_{i=1}^n \mathbb{E}(u^i) = v \quad (\lambda) \end{cases}$$

3. **Stochastic approximation version of Uzawa algorithm** to solve the dual of (P_3)

Four basic assumptions

Assumption

1. *F_0 **is convex**.*

For any $i \in \{1, \dots, n\}$:

2. *μ_i σ_i are Lipschitz w.r.t. space variables and have linear growth.*
3. *F_i has polynomial growth: $|F_i(u^i, x^i)| \leq K(1 + \sup_{0 \leq t \leq T} |x_t^i|^p)$.*
4. *$G_i : (\nu, \omega) \in L^2(0,T) \times \mathcal{C}([0,T]) \mapsto F_i(\nu, X^{i,\nu}(\omega)) \in \bar{\mathbb{R}}$. is **strictly convex w.r.t. the first variable**.*

Example: *Affine dynamics and separable local costs w.r.t. control and state*

$$\begin{cases} \mu_i(t,z,x) &= \alpha_i(t)z + \beta_i(t)x + \gamma_i(t)\,, \\ \sigma_i(x,t) &= \xi_i(t)x + \theta_i(t)\,, \\ F_i(\nu, X) &= g_i(\nu) + h_i(X) \end{cases}$$

where $\begin{cases} g_i : L^2(0,T) \to \mathbb{R} \text{ and } h_i : \mathcal{C}[0,T] \to \mathbb{R} \text{ are convex} \\ g_i \text{ or } h_i \text{ is strictly convex.} \end{cases}$

Approximating (P_1) by (P_2) $\rightarrow$ decentralized controls

Proposition *(Wellposedness)*

Both problems (P_1) and (P_2) admit a unique solution.

Let u^* denote the unique solution of (P_1)
and $\tilde{u}$ the unique solution of (P_2).
Let $\mathcal{F}^i$ denote the filtration generated by the local noise W^i.

Proposition *(Decentralized controls for (P_2))*

$\tilde{J}$ reaches its minimum over $\mathcal{U}$ at a unique point, $\tilde{u} \in \mathcal{U}$, such that for any i, $\tilde{u}^i$ is $\mathcal{F}^i$−adapted.

$\Rightarrow$ For any $j \neq i$, $\tilde{u}^i$ and $\tilde{u}^j$ are mutually independent. In particular

$$\frac{1}{n}\sum_{i=1}^{n} \tilde{u}^i \approx \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[\tilde{u}^i]$$

Let $\hat{\mathcal{U}} := \{u \in \mathcal{U} \,|\, u^i \text{ is } \mathcal{F}^i - \text{adapted for all } i \in \{1, \dots, n\}\}$

$$\min_{u \in \hat{\mathcal{U}}} \tilde{J}(u) = \min_{u \in \mathcal{U}} \tilde{J}(u).$$

Approximating (P_1) by (P_2) : ε-optimality

Theorem *(ε-optimality under some regularity assumptions on F_0)*

- ***If F_0 is Lipschitz with constant γ***, *then $\tilde{u}$ is an ε-optimal solution of (P_1), with $\varepsilon = \gamma M \sqrt{T/n}$.*
- ***If F_0 is Gâteaux differentiable with c-Lipschitz derivative***, *then $\tilde{u}$ is an ε-optimal solution of (P_1) with $\varepsilon = cTM^2/n$.*

Besides $J(\tilde{u}) - \tilde{J}(\tilde{u}) \geq J(\tilde{u}) - J(u^*) \geq 0$.

- **Ideas**: $0 \underbrace{\leq}_{\text{Jensen}} \mathbb{E}[F_0(\frac{1}{n}\sum_{i=1}^{n} \tilde{u}^i)] - F_0(\frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[\tilde{u}^i]) \underbrace{\leq}_{\substack{F_0 \text{ regularity} \\ \text{decentralized } \tilde{u}}} \varepsilon.$

For any $u' \in \mathcal{U}$ we have:

$$J(\tilde{u}) \underbrace{\leq}_{\substack{F_0 \text{ regularity} \\ \text{decentralized } \tilde{u}}} \tilde{J}(\tilde{u}) + \varepsilon \underbrace{\leq}_{\text{optimality of } \tilde{u}} \tilde{J}(u^{'}) + \varepsilon \underbrace{\leq}_{\text{Jensen}} J(u^{'}) + \varepsilon.$$

Lagrangian decomposition of $(P_3) \Leftrightarrow (P_2)$

- **Strong duality holds for (P_3)**

$$(P_3) \begin{cases} \min\limits_{u\in\mathcal{U}, v\in\mathcal{V}} \bar{J}(u,v), \quad \text{with} \quad \bar{J}(u,v) := F_0(v) + \dfrac{1}{n}\mathbb{E}\left(\displaystyle\sum_{i=1}^{n} F_i(u^i, X^{i,u^i})\right) \\ \text{s.t } \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}\mathbb{E}(u^i) - v = 0 \quad (\lambda) \end{cases}$$

- **The *Lagrangian* function** associated with (P_3) is:

$L : \mathcal{U} \times L^2(0,T) \times L^2(0,T) \to \bar{\mathbb{R}}$ defined by:

$$L(u,v,\lambda) := \bar{J}(u,v) + \langle \lambda, \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}(u^i) - v\rangle_{L^2(0,T)}. \tag{2}$$

- **The dual problem** (D) associated with (P_3) is:

$$(D) \quad \max_{\lambda\in L^2(0,T)} \mathcal{W}(\lambda), \quad \text{where } \mathcal{W}(\lambda) := \min_{u\in\mathcal{U}, v\in\mathcal{V}} L(u,v,\lambda). \tag{3}$$

with $\mathcal{V} := \{\nu \in L^2(0,T); |v(t)| \le 2M\,, \quad \forall t \in [0,T]\}$.

Uzawa algorithm: from *standard* to *sampled* algorithm

- **Maximizing the dual function** $\max_{\lambda \in L^2(0,T)} \mathcal{W}(\lambda)$,

$$\max_{\lambda \in L^2(0,T)} \min_{u \in \mathcal{U}, v \in \mathcal{V}} \left\{ F_0(v) + \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[F_i(u^i, X^{i,u^i})] + \langle \lambda, \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}(u^i) - v \rangle_{L^2(0,T)} \right\}.$$

- **Standard Uzawa algorithm** given steps $(\rho_k)_{k \geq 0}$ $\rho_k > 0$,

Update $\lambda^k \to \lambda^{k+1}$ along *dual iterations* $k \to k+1$

$$\begin{cases} v(\lambda^k) & \in & \arg\min_{v \in \mathcal{V}} \{ F_0(v) - \langle \lambda^k, v \rangle_{L^2(0,T)} \} \\ u^i(\lambda^k) & := & \arg\min_{u^i \in \hat{\mathcal{U}}_i} \left\{ \mathbb{E} \left(F_i(u^i, X^{i,u^i}) + \langle \lambda^k, u^i \rangle_{L^2(0,T)} \right) \right\} \\ Y(\lambda^k) & := & \mathbb{E}\left[\frac{1}{n} \sum_{j=1}^{n} u^j(\lambda^k) - v(\lambda^k) \right] \\ \lambda^{k+1} & = & \lambda^k + \rho_k Y(\lambda^k) \end{cases}$$

- **Sampled Uzawa algorithm** $Y(\lambda^k) \to \hat{Y}^{k+1}$:

$$\lambda^{k+1} = \lambda^k + \rho_k \hat{Y}^{k+1}$$

$$\text{where} \quad \hat{Y}^{k+1} = Y(\lambda^k) + r^{k+1} \quad \text{with} \quad \mathbb{E}[r^{k+1}] = 0 \; .$$

A natural choice for random variable $\hat{Y}^k$

- Along *dual iterations* $(\hat{Y}^1, \cdots, \hat{Y}^k, \cdots)$ are generated independently
- Choose a parameter $m \in \mathbb{N}^*$
- At iteration k
  - Generate independently from the past $I^{k+1} := (I^{1,k+1}, \dots, I^{m,k+1})$ m i.i.d r.v. uniformly in $\{1, \dots, n\}$
  - Compute the strategies $\left(u^{I^{1,k+1}}(\lambda^k), \cdots, u^{I^{m,k+1}}(\lambda^k)\right)$
  - Generate independently from the past $B^{k+1} := (B^{1,k+1}, \dots, B^{m,k+1})$ m i.i.d BM on $[0, T]$
  - Compute the decisions realization on B^{k+1}
  $$\left(u^{I^{1,k+1}}(\lambda^k)(B^{1,k+1}), \cdots, u^{I^{m,k+1}}(\lambda^k)(B^{m,k+1})\right)$$
  - $\hat{Y}^{k+1} := \dfrac{1}{m} \displaystyle\sum_{j=1}^{m} u^{I^{j,k+1}}(\lambda^k)(B^{j,k+1}) - v(\lambda^k)$
- One easily check that $\hat{Y}^{k+1} = Y(\lambda^k) + r^{k+1}$ with

$$\mathbb{E}[r^{k+1} \,|\mathcal{G}_k] = 0 \ , \quad \mathbb{E}[|r^{k+1}|^2 \,|\mathcal{G}_k] \leq C < \infty \quad \text{where } \mathcal{G}_k := \sigma\big((I^p, B^p)_{0 \leq p \leq k}\big) \ .$$

which constitutes useful properties for proving convergence...

Sampled Uzawa algorithm

Initialization

- $(\rho_k)_k \quad s.t. \quad \rho_k > 0, \quad \sum_{k=1}^{\infty} \rho_k = \infty, \quad \sum_{k=1}^{\infty} (\rho_k)^2 < \infty$
- $m \in \mathbb{N}^*$
- $k^{max} \in \mathbb{N}^*$
- $k = 0$
- $\lambda^0 \in L^2(0,T)$

While $k < k^{max}$

1. $v^k := v(\lambda^k) \in \arg\min_{v \in \mathcal{V}} \{F_0(v) - \langle \lambda^k, v \rangle_{L^2(0,T)}\}$
2. Generate independently also independently with previous r.v. $(I^p, B^p)_{1 \le p \le k}$
   - $I^{k+1} := (I^{1,k+1}, \ldots, I^{m,k+1}) \quad m$ i.i.d r.v. uniformly in $\{1, \ldots, n\}$
   - $B^{k+1} := (B^{1,k+1}, \ldots, B^{m,k+1})$ m i.i.d. BM on [0,T]
3. Compute $u^{I^{j,k+1}}(\lambda^k)$ where $u^i(\lambda) := \arg\min_{u^i \in \hat{\mathcal{U}}_i} \left\{\mathbb{E}\left(F_i(u^i, X^{i,u^i}) + \langle \lambda, u^i \rangle_{L^2(0,T)}\right)\right\}$
4. Compute the related control realizations $\left(u^{I^{1,k+1}}(\lambda^k)(B^{1,k+1}), \cdots, u^{I^{m,k+1}}(\lambda^k))(B^{m,k+1})\right)$
5. $\hat{Y}^{k+1} := \frac{1}{m} \sum_{j=1}^{m} u^{I^{j,k+1}}(\lambda^k)(B^{j,k+1}) - v(\lambda^k)$
6. $\lambda^{k+1} := \lambda^k + \rho_k \hat{Y}^{k+1}$
7. $k \leftarrow k + 1$

Convergence

Theorem *(Sampled Uzawa Algrithm and Decentralized Approach)*

1. $\mathcal{W}(\lambda^k) \xrightarrow[k\to\infty]{} \max_{\lambda\in L^2(0,T)} \mathcal{W}(\lambda)$ *a.s.*
2. $\{\lambda^k\} \rightharpoonup$ *to some* $\bar{\lambda} \in S := \arg\max_{\lambda\in L^2(0,T)} \mathcal{W}(\lambda)$ *in* $L^2(0,T)$ *a.s.*

If F_0 ***is Gâteaux diff. with c-Lipschitz derivative***, *then* $S = \{\bar{\lambda}\}$,

1. $\{\|\lambda^k - \bar{\lambda}\|^2_{L^2(0,T)}\} \to 0$ *a.s.*
2. $\{u(\lambda^k)\} \rightharpoonup u(\bar{\lambda}) = \tilde{u} := \arg\min_{u\in\mathcal{U}} \tilde{J}(u)$ *a.s.*

If moreover F_0 ***is strictly convex***, *then:*

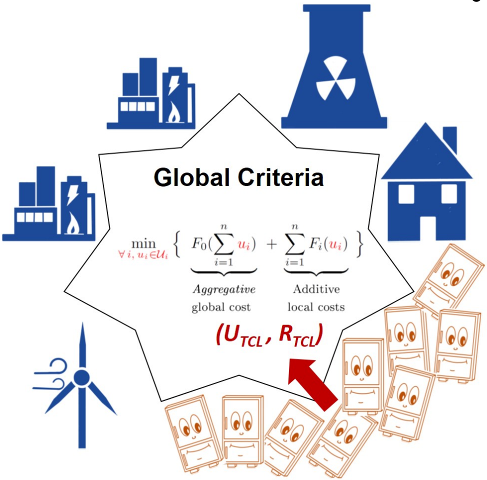
1. $\tilde{J}(u(\lambda^k)) \xrightarrow[k\to\infty]{} \tilde{J}(\tilde{u})$ *a.s.*
2. $\limsup_{k\to\infty} J(u(\lambda^k)) \leq \inf_{u\in\mathcal{U}} J(u) + 2\,\varepsilon(n)$ *a.s. with* $\varepsilon(n) = 2cTM^2/n$.

Proof *results for stochastic approximation in Hilbert spaces are sparse* $\Rightarrow$ *the convergence analysis was conducted inspired by [GeiersbachEtal19].*

Outline

EDF R&D

Thermostatically Controlled Loads (TCLs) providing Frequency Reserve [TindemansEtal15, BusicEtal16, DePaolaEtal19]



- **Global cost**: $F_0(U_{TCL}, R_{TCL})$
  - U_{TCL}: average consumption profile of TCLs
  - R_{TCL}: average Frequency Reserve provided by TCLs
  - F_0 = *Balancing cost* resulting from an optimization problem determining generation scheduling decisions (power production and Frequency Reserve (FR)) in order to minimize the short term operating cost of the system while matching generation and (flexible and inflexible) demand.

Thermostatically Controlled Loads (TCLs): state and control model [KizilkaleEtal16]

- **State** X_t^{i,u^i} temperature $[°C]$ controlled by its power consumption u_t^i $[W]$ evolves according

$$\begin{cases} dX_t^{i,u^i} & = & -\frac{1}{\gamma_i}(X_t^{i,u^i} - X_{OFF}^i + \zeta_i u_t^i)dt + \sigma_i \, dW_t^i, \quad \text{for } t \in [0,T], \\ X_{0,u^i}^i & = & x_0^i \in \mathbb{R}, \end{cases} \tag{4}$$

where:

- γ_i is its thermal time constant $[s]$.
- X_{OFF}^i is the ambient temperature $[°C]$.
- ζ_i is the heat exchange parameter $[°C/W]$.
- σ_i is a positive constant $[(°C)s^{\frac{1}{2}}]$,
- W^i is a Brownian Motion $[s^{\frac{1}{2}}]$, independent from W^j for any $j \neq i$.

- **Set of admissible controls**

$$\mathcal{U}_i := \{\nu \in H_i \text{ and } \nu_t(\omega) \in \{0, P_{ON,i}\} \ \text{ for a.e. } (t,\omega) \in [0,T] \times \Omega^i\}.$$

High dimensional stochastic control problem

- $U_{TCL} = \frac{1}{n}\sum_{i=1}^n u^i$ and $R_{TCL} = \frac{1}{n}\sum_{i=1}^n r_i(u^i, X^{i,u^i})$

$$(P_1^{TCL}) \quad \begin{cases} \inf\limits_{u\in\mathcal{U}} J(u) \\ J(u) := \quad \mathbb{E}\left(F_0\left(\frac{1}{n}\sum_{i=1}^n u^i, \frac{1}{n}\sum_{i=1}^n r_i(u^i, X^{i,u^i})\right)\right) \\ \qquad\qquad +\mathbb{E}\left(\frac{1}{n}\sum_{i=1}^n \int_0^T f_i(u_s^i, X_s^{i,u^i})ds + \gamma_i(X_T^{i,u^i} - \bar{X}^i)^2\right), \end{cases}$$

where, for any $i \in \{1, \ldots, n\}$ and any $s \in [0, T]$:

- Amount of FR provided by TCL i at time s:

$$r_i(u^i, X^{i,u^i})(s) := u_s^i \frac{X_s^{i,u^i} - X_{min}^i}{X_{max}^i - X_{min}^i}.$$

- Individual discomfort of TCL i at time s:

$$f_i(u_s^i, X_s^{i,u^i}) := \alpha_i\,(X_s^{i,u^i} - \bar{X}^i)^2 + \beta_i((X_{\min}^i - X_s^{i,u^i})_+^2 + (X_s^{i,u^i} - X_{\max}^i)_+^2).$$

- Terminal cost imposing periodic behavior, $\gamma_i(X_T^{i,u^i} - \bar{X}_i)^2$

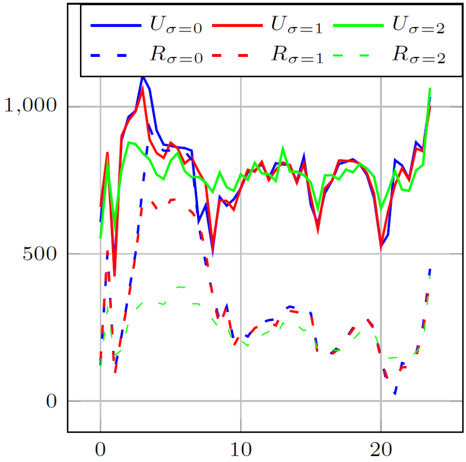
Simulation parameters [DePaolaEtal19]

- **Model parameters**
  - $T = 24$ hours
  - The generation technologies available in the system are
    - nuclear : installed capacity 10GW
    - Combined Cycle Gas Turbines (CCGT): installed capacity 25GW
    - Open Cycle Gas Turbines (OCGT): installed capacity 20GW
    - wind: installed capacity 40GW.
  - TCLs Population Size $n = 20 \times 10^6$
  - Power consumption $P_{ON,i} = 180W$, other parameters γ_i and X^i_{OFF} are taken from [?]
  - (Common) volatility of TCLs temperature dynamics: $\sigma_i := 0, 1, 2$
- **Sampled Uzawa Algorithm parameters**
  - $m = 317$ simulations per iteration of
  - Algorithm is stopped after $k^{max} = 75$ iterations
- **Local TCLs sub-problems** involve the *dual prices* (λ, ρ) associated to power and reserve constraints

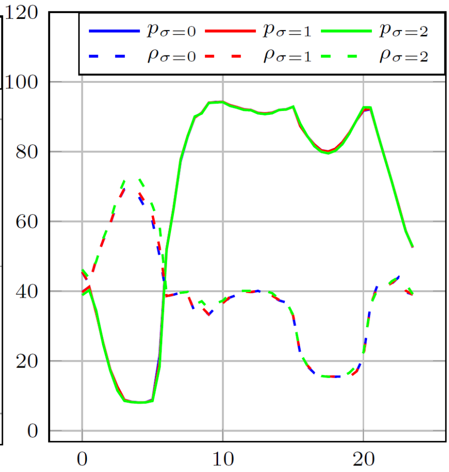
$$\inf_{u^i \in \mathcal{U}_i} \int_0^T f_i(u^i_s, X^{i,u^i}_s) + u^i_s \lambda^k_s - r_i(u^i, X^{i,u^i})(s) \rho^k_s \, ds,$$

are solved by a finite difference scheme the discretization discretizing the related one dimensional HJB equation with time steps $\Delta t = 7.6$ s and temperature steps $\Delta\theta = 0.15°$C.

TCLS Responses and Prices as a function of time (hours)

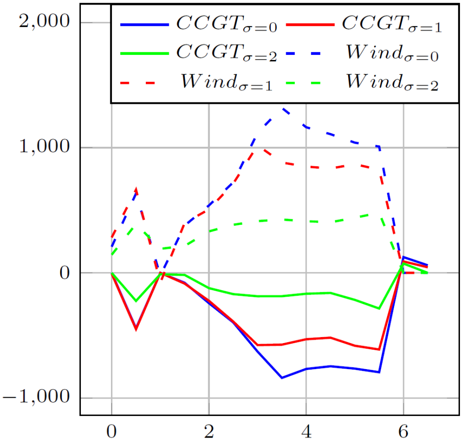


Power Consumption U (solid lines) (MWh) and Frequency Response R (doted lines) (MW)

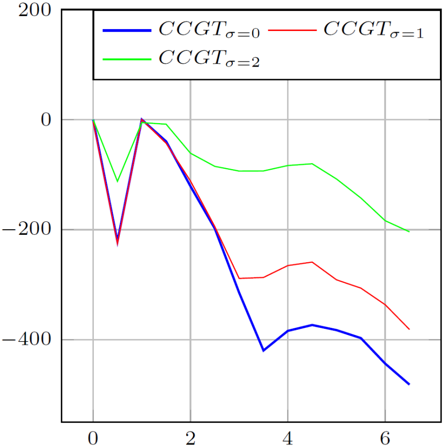


Power prices (£/MWh) and Reserve prices (£/MW)

Generators Power generation and Frequency Response deviations from *Buisness as Usual* as a function of time (hours)



Deviation of Power generation from Buisness as Usual (MWh)



Deviation of Frequency Response from Buisness as Usual (MW)

Minimized system costs in (£)

with or without TCL flexibilities

	$\sigma = 0$	$\sigma = 1$	$\sigma = 2$
BaU (M£)	27.70	27.70	27.72
Flex (M£)	27.19	27.25	27.40
BaU-Flex	-1.9%	-1.6%	-1.2%

Thank You !

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